**Theta Region Bowl Round #1**
**2008 FAMAT State Convention**

A. Suppose a quadrilateral ABCD with sides of length $2\sqrt{5}, 4\sqrt{5}, 6$ and 8 ( $AB = 2\sqrt{5}$, BC= $4\sqrt{5}$ , CD=6 and DA=8) is inscribed in a circle. What is the area of ABCD?
B. Given the same situation as above, what is $AC \cdot BD$ ?
C. If $x = \sqrt{30 - \sqrt{30 - \sqrt{30 - \cdots}}}$ , find $\lfloor \sqrt{x} \rfloor$, where $\lfloor x \rfloor$ represents the greatest integer less than or equal to x.
D. Let $f$ be an even function and $g$ an odd function, both with domain all real numbers. If the following table is given, find $(g \circ f^2)(-3) + (f \circ g)(-2)$, where $f^n(x) = (\overbrace{f \circ \cdots \circ f}^{\text{n times}})(x)$ .

| $x$ | 3 | 8 | 2 | 7 |
|---|---|---|---|---|
| $f(x)$ | -2 | -3 | -8 | -2 |
| $g(x)$ | 7 | 3 | -7 | -8 |

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**Theta Region Bowl Round #2**
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A. Let $A$ be the number of edges of a Mobius band, $B$ be the number of sides to a Mobius band, $C$ be the number of sides to a Klein bottle, and $D$ be the number of three dimensional faces on a hypercube. Find $(C+D)^{A+B}$.

B. Find the sum of the squares of the roots to $T(x)=3x^7-5x^6+4x^5-58x^2+87$.

C. $\sqrt{61-28\sqrt{3}}$ can be written in simplified radical form, $A+B\sqrt{C}$, where $A$, $B$ and $C$ are integers. Evaluate $\left(\frac{A}{C}\right)^B$.

D. Four men reach an old bridge in the middle of the night. Since the bridge is so old, a flashlight is needed to cross, but they have only one flashlight between them. Unfortunately, at most two people can cross at one time, and tossing the flashlight is not an option. One person takes 10 minutes to cross the bridge, another takes 5 minutes, another takes 2, and the last takes 1 minute to cross. If two people crossing the bridge can cross only as fast as the slowest person, what's the shortest amount of time to get all four across?

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**Theta Region Bowl Round #3**
**2008 FAMAT State Convention**

A. Let $M = \begin{bmatrix} -3 & 5 \\ -2 & 4 \end{bmatrix}$. Find $\det\left(\left(\left(M^{-1}\right)^T\right)^5\right)$.

B. If $f(x) = \dfrac{1}{x-1}$ and $g(x) = \dfrac{f(x+h) - f(x)}{h}$ for all $h$ not equal to 0, find a simplified expression for $g(2)$ in terms of h.

C. Indicate how many of the following statements are true:
  I. For any triangle T, we can find a circle C that is circumscribed by T.
  II. There are finitely many Pythagorean triples (not counting similar triangles, i.e., 3-4-5 and 6-8-10)
  III. If it is possible to square the circle with just a compass and straightedge, then it is possible to trisect an angle using only a compass and straightedge.

D. Given the function $\phi(x) = -5x^{13} - 4x^{12} - 3x^8 + 2x^3 - 8x^2 + kx - 2$ , where $k > 0$, let $a$ equal the smallest possible rational root of $\varphi$, let $b$ equal the largest possible quantity of negative real roots of $\varphi$, and let $c$ be the largest possible quantity of positive real roots of $\varphi$. Find the harmonic mean of $a$, $b$ and $c$.


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C. Indicate how many of the following statements are true:
  IV. For any triangle T, we can find a circle C that is circumscribed by T.
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  VI. If it is possible to square the circle with just a compass and straightedge, then it is possible to trisect an angle using only a compass and straightedge.

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**Theta Region Bowl Round #4**
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A. Let $Z(f)$ denote the set of all zeros of a function $f$. Find the sum of the absolute values of all elements of $Z(f)$ when $f(x)=x^6-1$.

B. Find the sum of all integral values of $x$ that satisfy the inequality $1<f(x)\le 11$ when $f(x)=\left(\sqrt{2}\right)^x$.

C. Find $7+\cfrac{1}{14+\cfrac{1}{14+\cfrac{1}{14+\cdots}}}$.

D. Find the area of the triangle with sides of lengths 8 and 9 with an angle of 60 degrees between them.

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**Theta Region Bowl Round #5**
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A. If $a$ is the product of the first 2 perfect numbers, $b$ is the sum of the first 3 abundant numbers, and $c$ is the sum of the first 7 odd primes, what is $a+b-c$ ? (A natural number $x$ is said to be abundant if the sum of its proper factors is greater than $x$.)
B. You have a beautiful solid-gold right-circular cone of height 36 inches with base circumference of $30\pi$ inches. Your arch-enemy steals the top of the cone (so that he now has a right circular cone), and tells you only that the volume of the part of the cone he stole is $100\pi$ cubic inches. What is the surface area of what he left you, in square inches?
C. Determine the sum of the coefficients of the polynomial obtained when we expand the expression $\left(4-3x-2x^2\right)^{11}$.
D. Let $\eta(x)=6x^2-38x$, with domain all real numbers. Find the sum of all values of $k$ for which $(28,k)$ is on the graph of the inverse relation of $\eta(x)$.

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**Theta Region Bowl Round #6**
**2008 FAMAT State Convention**

A. Find the complex coefficient of the term which contains $y^6$ in the expansion of $(2x-3iy^2)^7$. Of course, $i=\sqrt{-1}$.
B. Find the sum of the absolute value of all real or complex $\lambda$ that satisfy the equation $\det(A-\lambda I)=0$ where $A=\begin{bmatrix}5 & 4\\ 7 & 2\end{bmatrix}$ and $I$ is the identity matrix.
C. Find the length of the radius of a circle inscribed in an 8, 15, 17 triangle.
D. Consider the rhombus $ABCD$, and suppose $\overline{AC}$ has length 10 and $\overline{BD}$ has length 12. If $E$ is the point of intersection for $\overline{AC}$ and $\overline{BD}$, what is the area of $\Delta ABE$?

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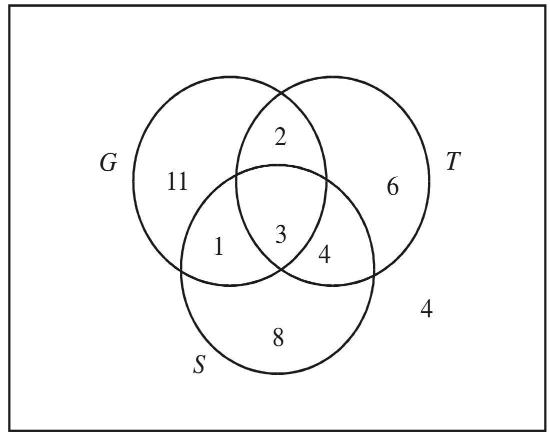
**Theta Region Bowl Round #7**
**2008 FAMAT State Convention**

A. Let $P$ be a regular nonagon. Find the sum of the measure of a single interior angle and the number of diagonals.

B. Find the sum of the squares of all values of $x$ which satisfy $x^2 - 6|x| - 7 = 0$.

C. Suppose the conic given by the equation $16x^2 + 9y^2 - 64x + 90y + 145 = 0$ and a circle of radius 7 share a single point of tangency. If $A$ is a point on the conic $16x^2 + 9y^2 - 64x + 90y + 145 = 0$ and $B$ is a point on the circle of radius 7, what is the maximum possible distance between $A$ and $B$?

D. The following diagram at right shows the number of junior students who play Golf, Tennis, and Squash at a local private school. How many people play exactly one sport, or no sport at all?




**Theta Region Bowl Round #7**
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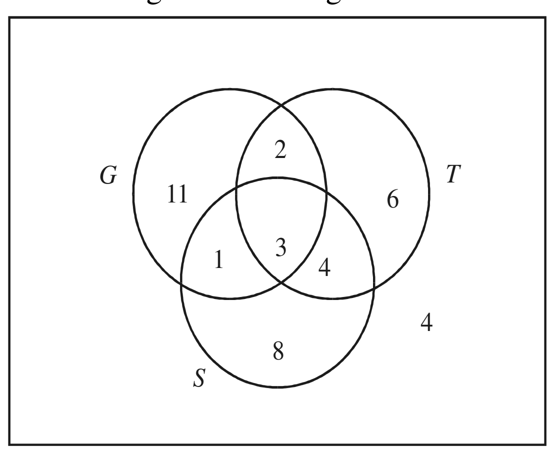

**Theta Region Bowl Round #8**
**2008 FAMAT State Convention**

A. Find the area of the conic given by the equation $4x^2+9y^2-4x+18y-26=0$.

B. Let the probability of event $A$ occurring be denoted by $P(A)$. If $P(A)=\frac{1}{2}$ and $P(B)=\frac{1}{3}$ and $A$ and $B$ are independent, calculate $P(A \text{ or } B)$.

C. Find the sum of the reciprocals of all roots of the function given by $m(x)=2x^3+4x^2-x-2$.

D. List the x-values of the corners of $f(x)=\left||x-4|-3\right|$.

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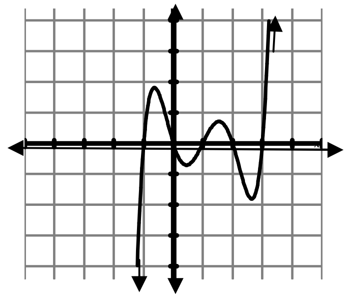
D. List the x-values of the corners of $f(x)=\left||x-4|-3\right|$.

**Theta Region Bowl Round #9** **2008 FAMAT State Convention**

A. Find the value of $x$ for which the matrix $A=\begin{bmatrix}3 & 2\\ x & -3\end{bmatrix}$ is idempotent. (A matrix is said to be idempotent when squaring the matrix results in the identity matrix.)

B. Let $f(x)$ be a polynomial function whose graph is shown below. The graph crosses the x-axis no more than five times. If n is the degree of this function then what is the minimum value of n?



C. Write $\frac{1}{4}$ as a decimal in base 3.

D. Find the sum of the absolute values of the y-values at which the graphs of $4x^2+3xy+4y^2=20$ and $f(x)=\frac{1}{x}$ intersect.

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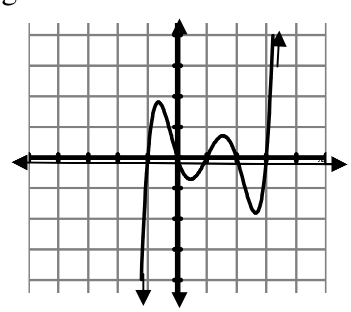

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**Theta Region Bowl Round #10**
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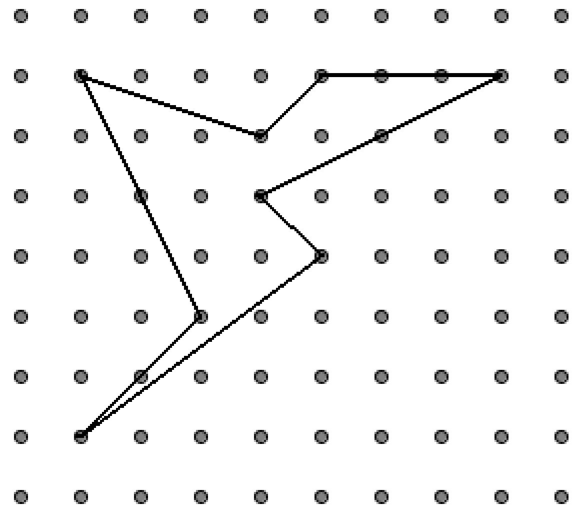
A. Find the area of the figure at right.
B. Find the semi-perimeter of the figure.
C. Find the area of the smallest convex polygon that encloses the figure.
D. Find the number of diagonals of the smallest convex polygon that encloses the figure.



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