1. A $x = \sqrt{2070 - x}$
   $0 = x^2 + x - 2070$
   $0 = (x-45)(x+46)$
2. E Use $u = \log_x 36$
   $2u^2 - 4u - 16 = 0$
   $2(u-4)(u+2) = 0$
   $\log_x 36 = 4, -2$

   $x = \sqrt{6},\ \frac{1}{6}$
3. B $3^{\log_7 49^{-2}} = 3^{(-2)\log_7 49} = 3^{-4} = \frac{1}{81}$
4. B $\lfloor \log 2008 + \log_{2008} 2008^{2008} \rfloor = \lfloor \log 2008 \rfloor + 2008 \log_{2008} 2008 = 3 + 2008 = 2011$
5. E The product contains $f(2007) = \log 1 = 0$, so the entire product is 0.
6. D $2008^{2008} = 251^{2008} \cdot 2^{3^{2008}}$
   # of divisors $= (2008+1) \cdot (3 \cdot 2008 + 1)$
   # of proper divisors $= (2008+1) * (3 * 2008 + 1) - 1 = 2009 * 3 * 2008 + 2008$
7. E $(\log_2(\log_3(\log_2 x)))^2 = 1$
   $\log_2(\log_3(\log_2 x)) = 1$ or $\log_2(\log_3(\log_2 x)) = -1$
   $\log_3(\log_2 x) = 2$ or $\log_3(\log_2 x) = \frac{1}{2}$
   $\log_2 x = 9$ or $\log_2 x = \sqrt{3}$
   $x = 2^9, 2^{\sqrt{3}}$
8. D
   $$\frac{\log 4}{\log 2} \cdot \frac{\log 5}{\log 3} \cdot \ldots \cdot \frac{\log 15}{\log 13} \cdot \frac{\log 16}{\log 14} = \frac{\log 15}{\log 2} \cdot \frac{\log 16}{\log 3} = 4 \cdot \log_3 15$$
9. A $\log_{d^2}(\sqrt[4]{p^7}) = \frac{\log p^{\frac{7}{4}}}{\log d^2} = \frac{7}{8}\frac{\log p}{\log d} = \frac{7}{8}\log_d p = 1757$
10. C $4^4 + 4^4 + 4^4 + 4^4 = 4 * 4^4 = 4^5 = 2^{10} = 1024$
11. B $\frac{251}{2008} = \frac{1}{8} = 3$ half-lives in 12 hours; half-life = 4 hours
12. A $\log_{\sqrt{2}}(x-2) + \log_{\sqrt{2}}(x-6) = \log_{\sqrt{2}}(x-2)(x-6) = 10$
    $(x-2)(x-6) = \sqrt{2}^{10} = 2^5 = 32$
    $x^2 - 8x - 20 = (x-10)(x+2) = 0$
    $(x-2), (x-6) > 0 \rightarrow x = 10$
13. A $\ln(e \cdot \sqrt{e} \cdot \sqrt[4]{e} \cdot \sqrt[8]{e} \cdot \ldots) = \ln e + \ln\sqrt{e} + \ln\sqrt[4]{e} + \cdots$

    $$= \ln e + \frac{1}{2}\ln e + \frac{1}{4}\ln e + \cdots$$

    $$= \left(\ln e)(1 + \frac{1}{2} + \frac{1}{4} + \cdots\right) = (1)(2) = 2$$
14. D $2008 - x^3 > 0 \rightarrow x < \sqrt[3]{2008}$ (x can be negative, so an infinite number of values)
15. B $6 = e^{a+b}$
    $\ln\frac{1}{24} = -\ln 24 = -(\ln 2 + \ln 2 + \ln 2 + \ln 3) = -(3a + b)$

16. B $(x^2-9x+19)^{x^2-8x+12}=1$ iff

(a) $x^2-9x+19=1$,

(b) $x^2-8x+12=0$, or

(c) $x^2-9x+19=-1$ and $x^2-8x+12$ evaluates to an even number

For (a), $x^2-9x+18=0 \to x=3,6$

For (b), $x^2-8x+12=0 \to x=2,6$

For (c), $x^2-9x+20=0 \to x=4,5$

the exponent $x^2-8x+12$ has an even value for $x=4$, odd value for $x=5$

$x=2,3,4,6$ are valid solutions, sum to 15

17. D $2^x=(2^3)^{y+5}$ and $(3^4)^x=(3^3)^{5-y}$

$x=3(y+5)$ and $4x=3(5-y) \to x-y=6-(-3)=9$

18. B Use $u=e^x$. $u+\frac{243}{u}=36$

$u^2+243=36u$

$e^x=u=27,9 \to \ln 27-\ln 9=\ln 3$

19. C $\frac{8^{m+3}-2^{3m+6}}{2^{3m+4}}=\frac{(2^3)^{m+3}}{2^{3m+4}}-\frac{2^{3m+6}}{2^{3m+4}}=2^5-2^2=28$

20. B $(5^{-1}-4^{-1})^{-1}=(\frac{1}{5}-\frac{1}{4})^{-1}=(-\frac{1}{20})^{-1}=-20$

21. D $f(x)=a\log_b x \to f(xy)=a\log_b(xy)=a\log_b x+a\log_b y=f(x)+f(y)$

22. A $i^{2009}+i^{2008}-i^{2007}=i+1-(-i)=1+2i$

23. B $\sqrt[3]{2x^2+3x-2}>0 \to 2x^2+3x-2>0 \to (x+2)(2x-1)>0$

Domain = $(-\infty,-2)\cup(\frac{1}{2},\infty)$

24. D $f(x)=a\cdot b^x$

$f(0)=a=4$

$f(3)=4\cdot b^3=108 \to b=3 \to f(x)=4\cdot 3^x \to f(2)-f(1)=36-12=24$

25. B $f(5)=3^5\cdot f(4)=3^5\cdot\left(3^4\cdot f(3)\right)=\cdots=3^{5+4+3+2+1}\cdot f(0)=3^{14}$

26. C (1)(0)+(9)(1)+(90)(2)+(900)(3)+(1)(4)=2893

27. D $\frac{5!}{4!\cdot 1!}(2^4)(-2^1)=-160$

28. B $3^{(2x^2)}=2^{(x-2)} \to$

$2x^2=\log_3 2^{(x-2)}=(x-2)\log_3 2$

$2x^2-(\log_3 2)x+2\log_3 2=0 \to$Product = ${}^{c}/_{a}=\log_3 2$

29. C At $x=2$, and $x=4$, also somewhere where $x<0$ ($x\approx -0.767$)

30. C $\log_x 2+\log_x 3+\cdots+\log_x 100=1$

$\log_x(2\cdot 3\cdot \ldots \cdot 100)=\log_x(100!)=1 \to x=100!$