For all questions, NOTA means none of the above answers is correct.

1. Find $\int_1^4 \frac{1-t}{\sqrt{t}}\ dt$

A. $-\frac{11}{3}$ B. $-\frac{8}{3}$ C. $\frac{14}{3}$ D. $\frac{20}{3}$ E. NOTA

2. The original version of Simpson's Rule is given by $\int_a^b f(x)\ dx \approx \frac{b-a}{6}\left[f(a)+4f\left(\frac{b-a}{2}\right)+f(b)\right]$.

Use Simpson's Rule to approximate $\int_0^4 (x^2-3x+6)\ dx$

A. $\frac{10}{3}$ B. $\frac{40}{3}$ C. $\frac{64}{3}$ D. $\frac{80}{3}$ E. NOTA

3. Find $\int_{\frac{\pi}{3}}^0 \cos^2\theta \sin\theta\ \mathrm{d}\theta$

A. $-\frac{7}{8}$ B. $-\frac{1}{8}$ C. $-\frac{7}{24}$ D. $\frac{1}{24}$ E. NOTA

4. $\int_0^1 x\sqrt{1-x}\ dx = \int_1^b (2-u)\sqrt{u-1}\ du$. Find b.

A. 0 B. 1 C. 2 D. 3 E. NOTA

For questions 5-7, define the integer integration operation as $i\int_a^b f(x)\ dx = \sum_{n=a}^{b} f(n)$.

5. Find $i\int_{-2}^{-1} x^x\ dx + i\int_1^2 x^x\ dx$.

A. $\frac{15}{4}$ B. $\frac{17}{4}$ C. $\frac{23}{4}$ D. $\frac{25}{4}$ E. NOTA

6. Find $i\int_{-3}^{3} x^2\ dx - \int_{-3}^{3} x^2\ dx$.

A. -10 B. 0 C. 10 D. 25 E. NOTA

7. Integer integration is equivalent to which of the following methods of approximation?

A. Left-hand Riemann B. Right-hand Riemann
C. Trapezoidal D. Simpson's Rule E. NOTA

8. Find the average value of $f(x) = \int_0^3 x\ dx$ on the interval $[0,3]$.

A. 1 B. $\frac{3}{2}$ C. 3 D. 9 E. NOTA

9. Find $\int_0^2 \ln(1+x^2)\ dx$

A. $2\ln 5-2$ B. $5\ln 5-5$ C. $2\ln 5-4+2\tan 2$ D. $2\ln 5-4+2\arctan 2$ E. NOTA

10. The shape bounded by the x-axis, the y-axis, the line $x = 4$, and the graph of $y = 4 - \sqrt{x}$ is rotated about the y-axis. What is the volume of the resulting solid?

A. $\frac{32}{5}\pi$ B. $\frac{88}{3}\pi$ C. $\frac{192}{5}\pi$ D. $\frac{1024}{5}\pi$ E. NOTA

11. Find $\int_{\frac{1}{10}}^{10} |\ln x| \, dx$

A. $\frac{99}{10}\ln 10 - \frac{81}{10}$ B. $\frac{101}{10}\ln 10 - \frac{99}{10}$ C. $\frac{99}{10} - \frac{101}{10}\ln 10$ D. 0 E. NOTA

12. Find $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n} \cos\left(\pi + \frac{1+i}{n}\right)$.

A. $\sin(\pi+1) - 1$ B. $\sin(\pi+1) + 1$ C. $-\sin 1$ D. $\frac{1}{2}\sin 2$ E. NOTA

13. $2\ln 2 + 75 = \int_a^b \left(1 + \frac{1}{x}\right) dx$. Given that a and b are integers, solve for a.

A. -75 B. 0 C. 25 D. 75 E. NOTA

14. $2\ln 2 + 75 = \int_a^b \left(1 + \frac{1}{x}\right) dx$. Which of the following could be a?

A. $-\infty$ B. -1000 C. -75 D. $-\frac{1}{2}$ E. NOTA

15. Let $f(x) = \sin x$ and $g(x) = \int_0^x f(t)\, dt$. Find the area between the graphs of $f(x)$ and $g(x)$ on the interval $\left[0, \frac{\pi}{2}\right]$.

A. 0 B. $2 - \frac{\pi}{2}$ C. $\frac{\pi}{2}$ D. $1 - \sqrt{2}$ E. NOTA

16. Kim is throwing a tea party for her friends from England. Here is how their tea cups are constructed: the graph of $y = 2x - x^2$ is rotated about the y-axis. The tea is poured in from above, along the y-axis. What volume of tea do each of these cups hold?

A. $\frac{\pi}{6}$ B. $\frac{5}{12}\pi$ C. $\frac{8}{15}\pi$ D. $\frac{5}{6}\pi$ E. NOTA

17. Find $\int \frac{\sin x \cos^2 x}{(\tan x)^3} \, dx$

A. $\frac{\sin^3 x}{3} - \csc x - 2\sin x + C$ B. $\frac{\cos^4 x - 4\sin^2 x + 4}{3\sin x} + C$ C. $-\frac{\cos^6 x}{6\sin x} + C$ D. $-\frac{\csc^4 x}{4} + C$ E. NOTA

18. It is common notation to let $\int_{(a,b)} f(x)\, dx = \int_a^b f(x)\, dx$. Find $\int_{\mathbb{R}} \frac{1}{1+x^2} \, dx$

A. 0 B. $\frac{\pi}{4}$ C. $\frac{\pi}{2}$ D. π E. NOTA

19. Find the average value of $f(x) = x^n$ on the interval $[0, n]$.

A. $\frac{n^n}{n+1}$ B. $\frac{n^{n+1}-1}{n+1}$ C. $\frac{n^{n+1}-1}{n(n+1)}$ D. $\frac{n^n}{n}$ E. NOTA

20. Archimedes' Spiral is given in polar form by the expression $r = \theta$. Find the area in Quadrant I bounded by Archimedes' Spiral on the interval $0 \le \theta < \frac{\pi}{2}$.

A. $\frac{\pi}{48}$ B. $\frac{\pi^2}{8}$ C. $\frac{\pi^3}{48}$ D. $\frac{\pi^3}{24}$ E. NOTA

21. Find $\sum_{k=1}^{n} \int_0^k \sqrt{kx}\, dx$

A. $\frac{3}{2}k^2$ B. $\frac{3}{2}nk^2$ C. $\frac{1}{9}n(n+1)(2n+1)$ D. $\frac{1}{9}n^2(n+1)(2n+1)$ E. NOTA

22. Find $\int\int x^{k+n}\, dk\, dn$

A. $\frac{x^{k+n}}{(\ln x)^2}+C_1n+C_2$ B. $\frac{x^k}{\ln x}+C_1n+C_2$ C. $\frac{x^{k+1}x^{n+1}}{(k+1)(n+1)}+C_1n+C_2$ D. $\frac{x^{k+n}}{(\ln(k+n))^2}+C_1n+C_2$ E. NOTA

23. Matt and Joker attack Ian with a jousting move that requires them to run with a velocity of $v(t) = \log(6t+3)$. How far do they run between $t = 0$ and $t = 10$?

A. $10-\frac{10}{\ln 10}$ B. $60\log 63-60+3\log 21$ C. $10\log 63-10+\frac{\log 21}{2}$ D. $10\log 63+\frac{\log 21}{2}-\frac{10}{\ln 10}$ E. NOTA

24. Let A be the parallelogram with vertices $[1,1]$, $[4,4]$, $[4,1]$, and $[7,4]$. What is the volume of the resulting solid if A is rotated about the x-axis?

A. 45π B. 46π C. $\frac{105}{2}\pi$ D. 63π E. NOTA

25. Find $\int_1^4 \frac{dx}{x^2+5x+4}$

A. $\ln 4$ B. $-\frac{4}{3}\ln 2 + \frac{2}{3}\ln 5$ C. $-\ln 8 + 2\ln 5 - \ln 2$ D. ∞ E. NOTA

26. Let $A = \sum_{n=2}^{\infty} \frac{\ln n}{n}$ and $B = \sum_{n=2}^{\infty} \frac{1}{n\ln n}$. Using the integral test, which of the following is true?

A. Both diverge B. Both converge C. Only A converges D. Only B converges E. NOTA

For questions 27 and 28, let $F_0 = 1$, $F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for all integers $n \geq 2$. You may assume that $\lim_{n\to\infty} \frac{F_{n+1}}{F_n} = \frac{\sqrt{5}+1}{2}$. Let $G_0 = \sin x$, $G_1 = \cos x$, and $G_n = G_{n-1} + G_{n-2}$ for all integers $n \geq 2$.

27. Find $\int_{-1}^{1} \frac{G_2}{G_1}\, dx$

A. 0 B. 2 C. $2 - 2\ln\cos 1$ D. $\sqrt{5} + 1$ E. NOTA

28. Find $\lim_{n\to\infty} \int_{2008}^{2010} \frac{G_{n+1}}{G_n}\, dx$

A. $\frac{\sqrt{5}-1}{2}$ B. 2 C. $\sqrt{5} + 1$ D. $2\sqrt{5} + 2$ E. NOTA

29. Let $[x]$ equal the largest integer less than or equal to x. Find $\int_0^{\pi} [x]\, dx$

A. $3\pi - 3$ B. $\pi + 3$ C. $3\pi - 6$ D. $4\pi - 6$ E. NOTA

30. Euler's sigma function is defined such that $\sigma(n)$ equals the sum of the integer divisors of n. Interestingly, Euler initially used the notation $\int n$ instead of $\sigma(n)$. Which of the following is true for Euler's sigma function given that p and q are prime?

A. $\int p \int q = \int (pq)$ B. $\int (p+q) = \int p + \int q$ C. $\int (p-q) = \int p - \int q$ D. $\int cq = c\int q$ E. NOTA