

- 1) Given the differential equation  $y'' = 20x^3 + 24x^2 + 2$  with  $y(0)=1$  and  $y'(0)=0$ . Find  $y(1)$ .  
 A) 1                      B) 3                      C) 5                      D) 7                      E) NOTA
  
- 2) Which of the following is a general solution to the differential equation  $ydx = \sin(x)dy$ ?  
 A)  $y = \tan\left(\frac{x}{2}\right) + C$                       B)  $y = C \tan\left(\frac{x}{2}\right)$                       C)  $y = e^{\cos(x)} + C$                       D)  $y = Ce^{\cos(x)}$   
 E) NOTA
  
- 3) Which of the following describes the differential equation  $(x^3 - x^2y + xy^2 - y^3)dx + (x^2 + y^2)dy = 0$ ?  
 A) Homogeneous of degree 1                      B) Homogeneous of degree 2                      C) Homogeneous of degree 3  
 D) Not homogeneous                      E) NOTA
  
- 4) Which of the following differential equation(s) is/are not exact?  
 I)  $x dx + y dy = 0$                       III)  $e^x dx + e^y dy = 0$   
 II)  $y dx + x dy = 0$                       IV)  $e^y dx + e^x dy = 0$   
 A) IV only                      B) II & IV                      C) III & IV                      D) All are exact                      E) NOTA
  
- 5) Solve the differential equation  $\frac{dy}{dx} = (x + y)^2$ ,  $y(0) = 0$ .  
 A)  $y = \frac{3}{x^3 + 3} - 1$                       B)  $y = \tan(x) - x$                       C)  $y = \frac{1}{x + 1} - x - 1$                       D)  $y = \tan^{-1}(x)$                       E) NOTA
  
- 6) Which of the following is a possible integrating factor,  $u(x)$ , for the differential equation  $x^2 dy = (1 - y \ln x) dx$ ?  
 A)  $u(x) = e^{-\frac{1}{x}} x^{-\frac{1}{x}}$                       B)  $u(x) = e^{-\frac{1}{x^2}}$                       C)  $u(x) = x^{-\frac{1}{x^2}}$                       D)  $u(x) = x e^{\frac{1}{x}}$                       E) NOTA
  
- 7) The population growth rate of Mu City is directly proportional to the population at any given time. Initially there are 20,000 Muans (people who live there) and after 1 year the population has tripled. After how many years will the population reach 1 million Muans?  
 A)  $\frac{\ln(50)}{3}$                       B)  $\ln\left(\frac{50}{3}\right)$                       C)  $\frac{\ln(50)}{\ln(3)}$                       D)  $\frac{50}{\ln(3)}$                       E) NOTA
  
- 8) Which of the following equations will satisfy the differential equation  $y^{(2008)} - y = 0$ ?  
 NOTE:  $y^{(2008)}$  is the 2008<sup>th</sup> derivative of  $y$ .  
 I)  $y = \sin(x)$                       II)  $y = \cos(x) - \sin(x)$                       III)  $y = e^x + 2\sin(x) - 3\cos(x)$                       IV)  $y = e^{-x} + e^x$   
 A) I only                      B) I & II                      C) I, II, & III                      D) All will satisfy                      E) NOTA
  
- 9) Which of the following differential equations is NOT an ordinary differential equation?  
 A)  $\frac{dy}{dx} = x^2 + 2$                       B)  $\frac{d^2y}{dx^2} = x^2 + 2$                       C)  $\left(\frac{dy}{dx}\right)^2 + y^2 = x^2 + 2$                       D)  $\left(\frac{dy}{dx}\right)^{\frac{1}{2}} + \sqrt{y} = x^2 + 2$                       E) NOTA

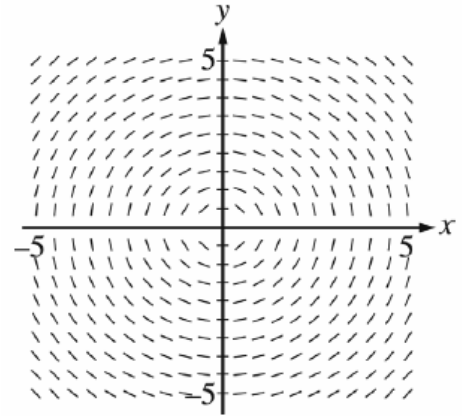
NOTA

10) Find the general solution to the differential equation  $y'' + 3y' + 2y = 0$ .

NOTE: In all answer choices  $C_1$  and  $C_2$  denote arbitrary constants.

- A)  $y = C_1e^{-x} + C_2e^{-2x}$       B)  $y = C_1xe^{-x} + C_2e^{-2x}$       C)  $y = C_1e^x + C_2e^{2x}$   
 D)  $y = C_1e^{3x} + C_2e^{2x}$       E) NOTA

11) Find the differential equation that could produce solutions that correspond to the solutions shown in the slope field diagram.



- A)  $xdx + ydy = 0$   
 B)  $xdx - ydy = 0$   
 C)  $ydx + xdy = 0$   
 D)  $ydx - xdy = 0$   
 E) NOTA

12) Given the initial value problem  $2x \frac{dy}{dx} = 3y$ ,  $y(0) = 0$ . Which of the following describes the solution(s),  $y = f(x)$ , to this initial value problem?

- A) There is a unique solution.  
 B) There are infinitely many solutions.  
 C) There is no solution.  
 D) More information is needed to draw a conclusion regarding the solution(s).  
 E) NOTA

13) Find the orthogonal trajectories of the family of curves represented by  $y = Ce^x$ , where  $C$  is any real number. NOTE: In all answer choices  $K$  denotes an arbitrary constant not necessarily equal to  $C$ .

- A)  $y = Ke^x$       B)  $y = Ke^{-x}$       C)  $y^2 = 2x + K$       D)  $y^2 = -2x + K$       E) NOTA

14) Given a general third order linear differential equation,  $ay''' + by'' + cy' + dy = 0$ . Two of the solutions are  $y = f(x)$  and  $y = g(x)$ ,  $f(x) \neq g(x)$ . Which of the following function(s) are also possible solutions?

- I)  $y = f(x) + g(x)$       II)  $y = f(x) - 3g(x)$       III)  $y = f(x)[1 + g(x)]$       IV)  $y = [1 - f(x)][1 - g(x)]$   
 V)  $y = [f(x)]^2 + [g(x)]^2$   
 A) I & II only      B) III & V only      C) I, II, & IV      D) I, II, III, & IV      E) NOTA

15) The acceleration of a particle moving along the  $x$ -axis is given by the equation  $a(t) = te^t - 2$  where time is measured in seconds and acceleration is measured in feet per second. Knowing that the particle initially starts at  $x(0) = 0$  feet and after 1 second the particle is at  $x(1) = 1$  foot, find the instantaneous velocity in feet per second at  $t = 1$  second.

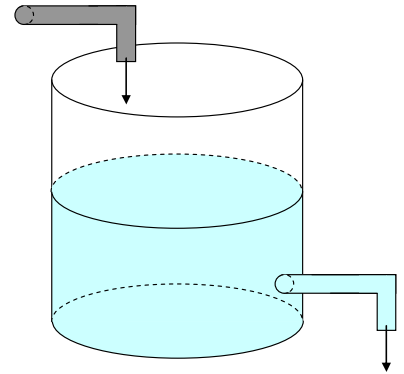
- A)  $-e - 2$       B) 0      C)  $e - 2$       D)  $e + 2$       E) NOTA

16) The function  $y = (2x + 3)e^x$  is a solution to the differential equation  $ay'' + by' + cy = 0$  where  $a$ ,  $b$ , and  $c$  are real numbers. Knowing this fact, what is/are the root(s) of the characteristic equation for this differential equation?

- A) {2,3}      B) {-2,-3}      C) {1,2}      D) {1}      E) NOTA

- 17) Given the differential equation  $y' = xy + 1$  with the initial condition  $y(0) = 1$  and step size  $\Delta x = 1$ . Approximate  $y(4)$  using Euler's method.  
 A) 49      B) 65      C) 206      D) 326      E) NOTA

- 18) A tank initially contains 100 gallons of pure water. An oil truck pulls up to the tank and pours a liquid that is 50% water and 50% oil into the tank at a rate of 10 gallons per minute. The tank, however, is leaking liquid out at a rate of 10 gallons per minute. If the tank starts leaking the second the truck starts pouring in its mixture, how many gallons of oil will be in the tank after 10 minutes?  
 A)  $50e^{-1}$       B)  $50(1 - e^{-1})$       C)  $100e^{-1}$   
 D)  $100(1 - e^{-1})$       E) NOTA



- 19) The half-life of Mu-42 is thirty seconds. If we start out with 64 grams of Mu-42, how many grams will remain after exactly 3 minutes?  
 A) 2      B) 4      C) 6      D) 8      E) NOTA
- 20) Which of the following integrating factors will make the differential equation  $\frac{dy}{dx} = e^x + y - 1$  exact?  
 A)  $u(x) = x$       B)  $u(x) = \frac{1}{x}$       C)  $u(x) = e^x$       D)  $u(x) = e^{-x}$       E) NOTA
- 21) Which of the following differential equation(s) is/are NOT linear?  
 I)  $y'' + 2y' + y = 0$   
 II)  $y'' + (y')(y) = 3$   
 III)  $e^{2x}y'' + (\sin x)y = \tan^{-1}(x)$   
 IV)  $(y'')^2 - y' + \frac{x}{y} = 6e^x$   
 A) IV only      B) I & III only      C) II & IV only      D) II, III, & IV      E) NOTA
- 22) Find the general solution to  $e^{2x} dx - (\sqrt{e^x - 1}) dy = 0, x > 0$   
 A)  $y = \frac{2}{3}\sqrt{e^x - 1}(e^x + 2) + C$       B)  $y = \frac{2}{3}(e^x - 1)^{3/2} + C$       C)  $y = 2e^x\sqrt{e^x - 1} + C$   
 D)  $y = \frac{e^x}{3}\sqrt{e^x - 1} + C$       E) NOTA
- 23) Ben decides to place a hot cup of coffee initially at 150°F into a 50°F cooler. After 30 minutes of being in the freezer, the temperature of the coffee has dropped to 120°F. If the temperature of the coffee continues to drop following Newton's Law of Cooling, what would be the temperature of the coffee after 1 hour?  
 A) 90°F      B) 96°F      C) 99°F      D) 100°F      E) NOTA

24) Given that  $2y'' = 3y^2$  under the initial conditions  $y(0)=1$  and  $y'(0)=-1$ , find  $y(1)$ .

- A)  $\frac{3}{2}$       B) 2      C) 3      D) 4      E) NOTA

25) Find the general solution to  $y'' = 24x + 12 - \frac{1}{x^2}$ .  $C_0$ ,  $C_1$ , and  $C_2$  are arbitrary constants.

- A)  $y = x^4 + 2x^3 + x \ln(x) + C_2x^2 + C_1x + C_0$   
 B)  $y = 4x^3 + 6x^2 + \ln(x) + C_1x + C_0$   
 C)  $y = 24x^3 + 12x^2 - \ln(x) + C_0$   
 D)  $y = 12x^2 + 12x - \ln(x) + C_0$   
 E) NOTA

26) Water is flowing into a basin at a rate in gallons per minute given by the following function:

$$f(t) = \begin{cases} 2t, & \text{if } 0 \leq t < 1 \\ 2, & \text{if } 1 \leq t < 3 \\ t^2 - 7 & \text{if } 3 \leq t \end{cases}$$

If the basin is initially empty how many gallons are present in the basin after 4 minutes?

- A)  $\frac{16}{3}$       B)  $\frac{20}{3}$       C)  $\frac{31}{3}$       D) 9      E) NOTA

27) The equation  $P(x)y'' + Q(x)y' + R(x)y = 0$  is *exact* if it can be written in the form  $[P(x)y']' + [f(x)y]' = 0$  where  $f(x)$  is determined in terms of  $P(x)$ ,  $Q(x)$ , and  $R(x)$ . How many of the following differential equations are NOT exact?

- I)  $y'' + xy' + y = 0$       II)  $xy'' - (\cos x)y' + (\sin x)y = 0$       III)  $x^2y'' + xy' - y = 0$   
 A) 0      B) 1      C) 2      D) 3      E) NOTA

28) Given  $g(x) = x + e^x - \sin(x)$ . Find  $g(0) + g'(0) + g''(0) + g^{(3)}(0) + \dots + g^{(2008)}(0)$  where  $g^{(n)}(0)$  denotes the  $n^{\text{th}}$  derivative of  $g(x)$  evaluated at  $x=0$ .

- A) 2007      B) 2008      C) 2009      D) 2010      E) NOTA

29) The population of a bacteria sample,  $P(t)$ , is given by the differential equation  $\frac{dP}{dt} = k(M - P)$  where  $k$  and  $M$  are constants. If  $P(0)=0$ , which of the following is a solution for  $P(t)$ ?

- A)  $P(t) = \frac{1}{M}e^{-kt}$       B)  $P(t) = M - \frac{1}{M}e^{kt}$       C)  $P(t) = M(1 - e^{-kt})$       D)  $P(t) = e^{kt} + M$       E) NOTA

30) How many of the following differential equations are separable?

- I)  $\frac{dy}{dx} = xy$       II)  $\frac{dy}{dx} = x + y$       III)  $\frac{dy}{dx} = \left(\frac{y-1}{x-1}\right)^2$       IV)  $\frac{dy}{dx} = (x-y)^2$   
 A) 0      B) 1      C) 2      D) 3      E) NOTA