1) $y' = 5x^4 + 8x^3 + 2x + C_1 \quad y'(0) = 0 \Rightarrow C_1 = 0$
$y = x^5 + 2x^4 + x^2 + C_2 \quad y(0) = 1 \Rightarrow C_2 = 1$; $y(1) = 5$ **C**

2) $\dfrac{dy}{y} = \dfrac{dx}{\sin x} \quad \ln y = \ln\left[\tan\left(\dfrac{x}{2}\right)\right] + C \Rightarrow y = C\tan\left(\dfrac{x}{2}\right)$ **B**

3) $x^3 - x^2y + xy^2 - y^3$ is homogeneous of degree 3 since
$(tx)^3 - (tx)^2(ty) + (tx)(ty)^2 - (ty)^3 = t^3(x^3 - x^2y + xy^2 - y^3)$
$x^2 + y^2$ is homogeneous of degree 2 since
$(tx)^2 + (ty)^2 = t^2(x^2 + y^2)$.
But since the two terms are not of the same degree, the differential equation is not homogeneous. **D**

4) $M(x, y)dx + N(x, y)dy$ is exact if $\dfrac{dM}{dy} = \dfrac{dN}{dx}$.

I) $\dfrac{dM}{dy} = 0 \quad \dfrac{dN}{dx} = 0$ II) $\dfrac{dM}{dy} = 1 \quad \dfrac{dN}{dx} = 1$ III) $\dfrac{dM}{dy} = 0$
$\dfrac{dN}{dx} = 0$ IV) $\dfrac{dM}{dy} = e^y \quad \dfrac{dN}{dx} = e^x$;
IV is only one that is not exact **A**

5) Let $v = x + y$; $\dfrac{dv}{dx} = 1 + \dfrac{dy}{dx} = 1 + v^2$. $\dfrac{dv}{1+v^2} = dx$,
$\tan^{-1}(v) = x + C \Rightarrow v = \tan(x + C) \Rightarrow x + y = \tan(x + C)$
$y = \tan(x + C) - x$. $y(0) = \tan(C) = 0 \Rightarrow C = 0$
$y = \tan(x) - x$ **B**

6) $\dfrac{dy}{dx} + \left(\dfrac{\ln x}{x^2}\right)y = \dfrac{1}{x^2} \quad P(x) = \left(\dfrac{\ln x}{x^2}\right)$, $u(x) = e^{\int P(x)dx}$
$\int\left(\dfrac{\ln x}{x^2}\right)dx = -\dfrac{1}{x}\ln x - \dfrac{1}{x}$ (using integration by parts)
$u(x) = e^{-\frac{1}{x}\ln x - \frac{1}{x}} = e^{-1/x}x^{-1/x}$ **A**

7) $\dfrac{dP}{dt} = kP \quad P(t) = Ce^{kt} \quad P(0) = 20000,\ P(1) = 60000$
$P(0) = C = 20000,\ P(1) = 20000e^k = 60000 \Rightarrow k = \ln(3)$
$1000000 = 20000e^{(\ln 3)t} \Rightarrow t = \dfrac{\ln(50)}{\ln(3)}$ **C**

8) Since 2008 is divisible by 4 and 2, the following are true: $\dfrac{d^{2008}}{dx^{2008}}(\sin x) = \sin x$; $\dfrac{d^{2008}}{dx^{2008}}(\cos x) = \cos x$;
$\dfrac{d^{2008}}{dx^{2008}}(e^x) = e^x$; $\dfrac{d^{2008}}{dx^{2008}}(e^{-x}) = e^{-x}$. Thus all 4 will satisfy the differential equation. **D**

9) All of the differential equations are ordinary differential equations because the dependent variable *y* is only dependent upon on variable *x*. **E**

10) The characteristic equation is $r^2 + 3r + 2 = 0$. The roots are $r = -1, -2$. The general solution is
$y = C_1e^{-x} + C_2e^{-2x}$ **A**

11) The slope field shows solutions that are circles centered at the origin $x^2 + y^2 = C$. The derivative is
$2xdx + 2ydy = 0 \Rightarrow xdx + ydy = 0$ **A**

12) $\dfrac{dy}{y} = \dfrac{3}{2}\dfrac{dx}{x} \Rightarrow \ln y = \dfrac{3}{2}\ln x + C \Rightarrow y = Cx^{3/2}$
$y(0) = C(0)^{3/2} = 0$ This is true for any value of *C*. Thus there are infinitely many solutions. **B**

13) $y = Ce^x$, $\dfrac{dy}{dx} = Ce^x = y$. To find the orthogonal trajectories, find the negative reciprocal of the slope:
$\dfrac{dy}{dx} = -\dfrac{1}{y}$, $ydy = -dx$, $\dfrac{1}{2}y^2 = -x + K \Rightarrow y^2 = -2x + K$ **D**

14) The solutions to a linear differential equation must be of the form $c_1f(x) + c_2g(x)$. I & II only **A**

15) $a(t) = te^t - 2$; $v(t) = (t-1)e^t - 2t + C_1$ (using integration by parts). $x(t) = (t-2)e^t - t^2 + C_1t + C_2$
$x(0) = (-2)e^0 - 0^2 + C_1(0) + C_2 = 0 \Rightarrow C_2 = 2$
$x(1) = (-1)e^1 - 1^2 + C_1(1) + 2 = 1 \Rightarrow C_1 = e$
$v(0) = (0)e^1 - 2 + e = e - 2$ **C**

16) If $y = (c_1x + c_2)e^{rx}$ is a solution to the differential equation $ay'' + by' + cy = 0$, then its characteristic equation will have a double root of *r*. In this problem the double root is 1. **D**

17) $y' = f(x, y) = xy + 1 \quad y_{n+1} = y_n + f(x_n, y_n) =$
$y_n + x_ny_n + 1 \quad y_1 = y_0 + x_0y_0 + 1 = 1 + (0)(1) + 1 = 2$
$y_2 = 2 + (1)(2) + 1 = 5, \quad y_3 = 5 + (2)(5) + 1 = 16$
$y_4 = 16 + (3)(16) + 1 = 65$ **B**

18) Let $x(t)$ be the number of gallons of oil in the tank at time *t* minutes. Initially $x(0) = 0$. The rate at which the amount of oil is changing is equal to the rate coming into the tank minus the rate coming out. Since 50% of the 10 gallons of liquid that is entering per minute is oil, then there is 5 gallons of oil entering per minute. The amount of oil leaving is dependent upon $x(t)$, which is $\dfrac{10x}{100} = \dfrac{x}{10}$.
Thus $\dfrac{dx}{dt} = 5 - \dfrac{x}{10} \Rightarrow \dfrac{dx}{50 - x} = \dfrac{dt}{10} \quad -\ln(50 - x) = \dfrac{t}{10} + C$
$x(t) = 50 - Ce^{-t/10}$, $x(0) = 50 - C = 0 \Rightarrow C = 50$
$x(0) = 50 - 50e^{-1} = 50(1 - e^{-1})$ **B**

19) $N(t)=N_0e^{-\lambda t}$, $N_0=64$, $\lambda=\frac{\ln(2)}{t_{1/2}}=\frac{\ln(2)}{0.5\text{ min}}=2\ln 2$

$N(3)=64e^{-6\ln 2}=64\left(\frac{1}{64}\right)=1$ **E**

20) $y'-y=e^x-1$ $u(x)=e^{\int P(x)dx}$ is an integrating factor that makes the differential equation $y'+P(x)y=Q(x)$ exact. For $P(x)=-1$, $u(x)=e^{-x}$ **D**

21) A linear differential equation must be linear in the dependent variable *y* and its derivatives y' and y''. It can be written as $A(x)y''+B(x)y'+C(x)y=F(x)$. II and IV do not satisfy this form. **C**

22) $\frac{dy}{dx}=\frac{e^{2x}}{\sqrt{e^x-1}}$ $u=e^x-1$, $du=e^x$, $e^x=u+1$

$\int\frac{e^{2x}dx}{\sqrt{e^x-1}}=\int\frac{e^xe^xdx}{\sqrt{e^x-1}}=\int\frac{(u+1)}{\sqrt{u}}du=\frac{2}{3}u^{3/2}+2u^{1/2}+C$

$=\frac{2}{3}\sqrt{u}(u+3)=\frac{2}{3}\sqrt{e^x-1}\left(e^x+2\right)$ **A**

23) $\frac{dT}{dt}=-k(T-T_s), T_s=50$ (surrounding temperature)

$\frac{dT}{T-50}=-kdt\Rightarrow\ln(T-50)=-kt+C$

Initial temp: $\ln(150-50)=-k(0)+C\Rightarrow C=\ln(100)$

$\ln(120-50)=-k(30)+\ln(100)\Rightarrow k=-\frac{1}{30}\ln\left(\frac{7}{10}\right)$

$\ln(T-50)=\frac{1}{30}\ln\left(\frac{7}{10}\right)(60)+\ln(100)\Rightarrow$

$\ln(T-50)=\ln\left(\frac{49}{100}\cdot 100\right)\Rightarrow T=49+50=99$ **C**

24) Let $u=\frac{dy}{dx}\Rightarrow\frac{du}{dx}=\frac{du}{dy}\frac{dy}{dx}=\frac{du}{dy}u$

$2\frac{du}{dy}u=3y^2\Rightarrow 2udu=3y^2dy\Rightarrow u^2=y^3+C_1$

Given $u(0)=-1$ and $y(0)=1\Rightarrow C_1=0$

$u^2=y^3\Rightarrow\frac{dy}{dx}=y^{3/2}\Rightarrow\frac{dy}{y^{3/2}}=dx\Rightarrow-\frac{2}{y^{1/2}}=x+C_2$

$y(0)=1\Rightarrow C_2=-2$ $y(1)=4$ **D**

25) $y'=12x^2+12x+\frac{1}{x}+C_1$,

$y=4x^3+6x^2+\ln(x)+C_1x+C_0$ **B**

26) $V(t)=\int_0^t f(u)du$

$V(4)=\int_0^1 2udu+\int_1^3 2du+\int_3^4\left(u^2-7\right)du=1+4+\frac{16}{3}=\frac{31}{3}$ **C**

27) $P(x)y''+P'(x)y'+f(x)y'+f'(x)y=0$

$P'(x)+f(x)=Q(x)\Rightarrow f(x)=Q(x)-P'(x)$

$f'(x)=R(x)$

Need to check that $R(x)$ is the derivative of $Q(x)-P'(x)$

I) $P(x)=1, Q(x)=x, R(x)=1$. $Q(x)-P'(x)=x$ The first derivative is 1, which is $R(x)$, IS EXACT

II) $P(x)=x, Q(x)=-\cos x, R(x)=\sin x$.

$Q(x)-P'(x)=-\cos x-1$ The first derivative is $\sin x$, which is $R(x)$, IS EXACT

III) $P(x)=x^2, Q(x)=x, R(x)=-1$. $Q(x)-P'(x)=-x$ The first derivative is –1, which is $R(x)$, IS EXACT

All are exact. **A**

28) $g(x)=x+e^x-\sin x$ $g'(x)=1+e^x-\cos x$

$g''(x)=e^x+\sin x$ $g^{(3)}(x)=e^x+\cos x$

$g^{(4)}(x)=e^x-\sin x$ $g^{(5)}(x)=e^x-\cos x$

$g^{(6)}(x)=e^x+\sin x$ $g^{(7)}(x)=e^x+\cos x$

$g(0)=1$ and $g'(0)=1$; $g^{(2n)}(0)=1$, $g^{(4n-1)}(0)=2$, $g^{(4n+1)}(0)=0$ for $n\geq 1$

$[1+1]+[1+2+1+0]+[1+2+1+0]+...+[1+2+1]=$

$=2010$ **D**

29) $\frac{dP}{M-P}=kdt\Rightarrow-\ln(M-P)=kt+C$ $-\ln(M)=C$

$\ln(M-P)=\ln(M)-kt\Rightarrow M-P=Me^{-kt}$

$P(t)=M\left(1-e^{-kt}\right)$ **C**

30) A separable D.E. takes the form $M(x)dx+N(y)dy=0$

I) Separable: $xdx-\frac{1}{y}dy=0$

II) Not separable

III) Separable: $\frac{1}{(x-1)^2}dx-\frac{1}{(y-1)^2}dy=0$

IV) Not separable. 2 are separable **C**