1. **B** $\lim_{x\to 5}\frac{2x^2-7x-15}{x^2-5}=\lim_{x\to 5}\frac{(2x+3)(x-5)}{(x+5)(x-5)}=\lim_{x\to 5}\frac{(2x+3)}{(x+5)}=\frac{2\cdot 5+3}{5+5}=\frac{13}{10}$.

2. **C** $f'(x)=e^x+2(2x+3)^1\cdot 2\Rightarrow f'(1)=e^1+4(2(1)+3)$.

3. **C** Let $u=\sin x$. Then $\int\cos(x)\cos(\sin x)\,dx=\int\cos u\,du=\sin u=\sin(\sin x)$.

4. **B** If we order the 8 points from least to greatest according to their x-coordinates, then the Mean Value Theorem guarantees at least one point x somewhere in between each pair of neighboring points where $f'(x)=4$ (the average rate of change of f on each interval, since both neighboring points are on $y=4x+3$.) There are 7 intervals, and it is fairly easy to draw such a curve with slope equal to 4 at exactly 7 points to show this minimum can happen.

5. **D** $\lim_{n\to\infty}\sum_{i=1}^{n}\left[\left(\frac{i}{n^2}\right)2^{\frac{3i}{n}+2}\right]=\lim_{n\to\infty}\sum_{i=1}^{n}\left[(\Delta x)x_n 2^{3x_n+2}\right]=\int_0^1 x2^{3x+2}dx$.

6. **B** $\lim_{n\to\infty}nd^2\sin\left(\frac{\pi}{n}\right)\cos\left(\frac{\pi}{n}\right)=\lim_{n\to\infty}\pi\frac{\sin\left(\frac{\pi}{n}\right)}{\pi/n}\cos\left(\frac{\pi}{n}\right)d^2=\pi(1)(1)d^2=\pi d^2$. Intuitively, we should get the area of a circle with radius d.

7. **A** We need the first derivative to be zero at 2 and 4, so $f'(x)=\pm(x-2)(x-4)$ are candidates. Also, we need $f''(2)<0$ and $f''(4)>0$, and only $f'(x)=-(x-2)(x-4)$ works now. Integrate it to find the polynomial in A.

8. **C** The smallest value of a should be when the line $y=ax$ is tangent to $y=e^x$. The derivative of e^x is just e^x, however, so we are requiring that $a=e^x$ at the appropriate value of x. But then $e^x=ax=e^x x\Rightarrow x=1$, so $a=e^1=e$.

9. **C** Find the area under the curve (which is just the top half of a circle) between 0 and x by drawing a straight line from the origin to the circle at x, dividing the region in question into a sector of the circle and a right triangle. The area of the triangle is $\frac{1}{2}x\sqrt{1-x^2}$ and the area of the sector is $\frac{\frac{\pi}{2}-\cos^{-1}x}{2\pi}(\pi\cdot 1^2)$.

10. **D** $\ln y=(x+2)\ln x\Rightarrow\frac{y'}{y}=\ln x+\frac{x+2}{x}\Rightarrow y'=x^{x+2}(\ln x+\frac{x+2}{2})$. At $x=2$, $y'=2^{2+2}(\ln 2+\frac{2+2}{2})=16(\ln 2+2)$.

11. **B** $v(t)=s'(t)=\frac{e^t}{1+e^t}\to\lim_{t\to\infty}v(t)=\lim_{t\to\infty}\frac{1+e^t-1}{1+e^t}=\lim_{t\to\infty}1-\frac{1}{1+e^t}=1$.

12. **D** Let $F(t)$ be an antiderivative of $\sin(t^2)$. Then $f(x)=\frac{d}{dx}[F(x^2)-F(0)]=F'(x^2)\cdot 2x=2x\sin(x^4)$. Then $f(2)=4\sin(16)$.

13. **A** If $g(x)$ satisfies $y'-xy=0$, then $f(x)+g(x)$ will still satisfy $y'-xy=x^2+2008$. Solving $y'-xy=0$, we have $\frac{1}{y}dy=xdx\Rightarrow\ln y=\frac{1}{2}x^2\Rightarrow y=e^{\frac{1}{2}x^2}$. Thus, $f(x)+e^{\frac{x^2}{2}}$ is also a solution.

14. **A** $V=\frac{4}{3}\pi r^3\Rightarrow dV/dr=4\pi r^2$. Then $dr/dt=(dV/dt)(1/(dV/dr))=1\cdot\frac{1}{4\pi r^2}$, so at $r=1$ the radius is decreasing at a rate of $1/4\pi$ *m/s*.

15. **B** Use the shell method: $2\pi\int_0^2 x(4-x^2)dx=2\pi\left[2x^2-\frac{1}{4}x^4\right]_0^2=8\pi$.

16. **E** B & C cannot work because $f'(x)>2$ for $5\le x\le 10$ implies $f(10)\ge 3+2\cdot 5=13$ and $f(10)\ge 1+2\cdot 5=11$, respectively, which contradicts $f(10)=10$. A & D don't work because the Mean Value Theorem says there is a point in the interval (0,5) where $f'(x)=\frac{5-0}{5-0}=1$ and $f'(x)=\frac{1-0}{5-0}=\frac{1}{5}>\frac{1}{10}$ and both of these contradict $f''(x)>0$ (since $f'(x)$ would not be able to be increasing).

17. **C** $x_1 = 0 - \frac{-1}{1} = 1,\ x_2 = 1 - \frac{1}{4} = \frac{3}{4}$.

18. **A** If *f* does not change sign at *x*, then there is an interval about *x* small enough such that either $|f(x)| = f(x)$ on the whole interval or $|f(x)| = -f(x)$ on the whole interval. In either case, the derivative of $|f(x)|$ is just $f'(x)$ or $-f'(x)$. However, if f does change sign at *x*, then on one side of *x* we will have $|f(x)| = f(x)$ and on the other side $|f(x)| = -f(x)$, so the left-hand derivative will be equal to the right-hand derivative if and only if $f'(x) = -f'(x) \Rightarrow f'(x) = 0$. Then for each of the functions we must check the points where they change sign and make sure their derivative is equal to zero. Only A satisfies this (since $f(x) = (x-2)^3 \Rightarrow f'(x) = 3(x-2)^2 \Rightarrow f'(2) = 0$ and the others have non-zero derivative where they cross the x-axis).

19. **B** $f'(x) = x^2 - 4x + 3 = (x-3)(x-1) \to$ Check $x = 2,3,4$. $f(3) = 1$ is the least.

20. **E** $a(t) = 20 \to v(t) = 20t$. Setting the velocity equal to 129, we find he will reach the required speed after $t = 129/20$ seconds. Then he should start at least $\int_0^{129/20} 20t\ dt = \frac{129^2}{200}$.

21. **C** For any partition of $[a,b]$, we can choose a point in every subinterval where *f* is 0, so that one Riemann sum for this partition is just 0. On the other hand, we can pick points in every subinterval where *f* is 1, so that another Riemann sum for the same partition yields $b-a$. But then the limit of all Riemann sums as the length of the largest subinterval goes to 0 cannot exist.

22. **A** The sum is $\sum_{i=1}^{2007} \log_{2008}(i+1) - \log_{2008} i = \log_{2008} 2008 - \log_{2008} 1 = 1 - 0 = 1$

23. **C** C implies that *f* must be increasing, and so it is 1-1. For counterexamples to A, B, and D, take $x^2, 0, e^x - x$.

24. **A** $g'(x) = e^{-x}(-F(x) + F'(x)) = e^{-x}\left(f'(x) + f''(x) + \ldots + f^{2008}(x) - f(x) - f'(x) - \ldots - f^{2008}(x)\right)$.

(Because $f^{2009}(x) = 0$.) But this last expression is just $-e^{-x} f(x)$, so $g'(1) = -\frac{11}{e}$.

25. **D** $f'(x) = -f'(-x) \to A \& B$. $f''(x) = -(-f''(-x)) = f''(-x) \to C$. $f = 0$ is a counterexample to D.

26. **A** $\int_{\pi/6}^{\pi/3} \ln(\tan x)dx = \int_{\pi/6}^{\pi/3} \ln\left(\cos\left(\frac{\pi}{2} - x\right)\right) - \ln(\cos(x))\,dx = \int_{\pi/3}^{\pi/6} -\ln(\cos(x))\,dx - \int_{\pi/6}^{\pi/3} \ln(\cos(x))\,dx =$

$\int_{\pi/6}^{\pi/3} \ln(\cos(x))\,dx - \int_{\pi/6}^{\pi/3} \ln(\cos(x))\,dx = 0$.

27. **D** $\frac{dy}{dt} = .1 - .1y \to \frac{1}{1-y} dy = .1dt \to -\ln(1-y) = .1t \to y = 1 - e^{-.1t}$. Plugging in $t = 10$ gives the answer.

28. **C** Differentiability implies continuity.

29. **C** By induction, $f^n(x) = xe^x + ne^x$, so $f^{2008}(1) = 1e^1 + 2008e^1 = 2009e$.

30. **D** The left-hand derivative is clearly 1. If $h > 0$ (and less than 1), then $\frac{f(0+h) - f(0)}{h} = \frac{1}{(n+1)h}$ where $\frac{1}{n+1} \le h < \frac{1}{n}$. So $\frac{n}{n+1} \le \frac{f(0+h) - f(0)}{h} = \frac{1}{(n+1)h} \le \frac{n+1}{n+1}$, and as $h \to 0$, we must have that $n \to \infty$, so by the squeeze theorem, $\lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = 1$. The left-hand and right-hand derivatives both equal 1, so *f* is differentiable at x = 0 and $f'(0) = 1$.