

1.  $A = \cos(\ln(e^0 + 0 + 0^2)) \frac{e^0 + 1 + 2 \cdot 0}{e^0 + 0 + 0^2} = 2$ .  $B = \int_0^\pi \frac{1 + \cos 2x}{2} dx = \frac{\pi}{2} + 0 = \frac{\pi}{2}$ .  $C = 0$  since

the function is odd.  $D = g'(0) = \frac{1}{(1-0)^2} = 1$ .  $A+B+C+D = \boxed{3 + \frac{\pi}{2}}$

2.  $A = \lim_{x \rightarrow 2} \frac{x+2}{x+4} = \frac{2}{3}$ .  $B = \lim_{x \rightarrow 0} \frac{7 \sin(7x)}{2 \cdot 7x} = \frac{7}{2} \cdot 1$ .  $C = \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 4x + 4}}{3x + 5} = \lim_{x \rightarrow \infty} \frac{x+2}{3x+5} = \frac{1}{3}$ .

$D = 0$ , since exponentials grow faster than polynomials.  $A+B+C+D = \boxed{\frac{9}{2}}$ .

3. The length the particle has traveled along the curve at time  $t$  is given by the arc length formula:  $l(t) = \int_0^{2t} \sqrt{1 + \cos^2(x)} dx$ . The particle's speed at time  $t$  is

$\frac{dl(t)}{dt} = 2\sqrt{1 + \cos^2(2t)}$ , and plugging in  $t = \frac{\pi}{2}$ , our answer is  $\boxed{2\sqrt{2}}$ .

4.  $A = \int_1^2 \frac{1}{x^2} dx = \frac{1}{2}$ .  $B = \int_0^1 \frac{2 \ln(x+1)}{x+1} dx = 2 \left[ \frac{1}{2} (\ln(x+1))^2 \right]_0^1 = (\ln 2)^2$ .  $C$  - Use implicit

differentiation to solve for  $y' = \frac{y}{2y-x}$  ...now plug in (1,-4) to get 4/9.  $D = 1$ , since

$y' = \frac{-2x}{(x^2+1)^2} e^{\frac{1}{x^2+1}}$  is only zero at  $x = 0$ .  $A + e^{\sqrt{B}} + C + D = \boxed{71/18}$

5.  $A = \int_0^1 x^3 - 3x^2 + 2x + 1 dx = \frac{1}{4} - 1 + 1 + 1 = \frac{5}{4}$ .  $B = C = 1$  since  $f'(x) = 3x^2 - 6x + 2$  has

two real roots (only true for a cubic).  $D = 1$  since  $f''(x) = 6x - 6$  has one real root.

$A+B+C+D = \boxed{17/4}$

6.  $A = \int_0^{\pi/4} \cos x - \sin x dx + \int_{\pi/4}^{5\pi/4} \sin x - \cos x dx + \int_{5\pi/4}^{2\pi} \cos x - \sin x dx$   
 $= (\sqrt{2} - 1) + (2\sqrt{2}) + (1 + \sqrt{2}) = 4\sqrt{2}$ .

$B = 2 \int_0^\pi \sin x dx = 4$   $C = 2 \int_{\pi/2}^{3\pi/2} -\cos x dx = 4$   $B+C-A = \boxed{8 - 4\sqrt{2}}$

7.  $A = \pi \int_0^1 x^2 - x^4 dx = \frac{2\pi}{15}$   $B = 2\pi \int_0^1 x(x-x^2) dx = \frac{\pi}{6}$   $A/B = \boxed{4/5}$

8.  $A = B = 0$  by simple algebra (no need to plug in values).  $C = 1$  since the Mean Value Theorem guarantees a point in the interval where  $h'(x) = 0$  since  $h(1) = h(2008) = 0$ , and this setting the derivative of  $h$  equals to zero yields the desired statement.  $A+B+C = \boxed{1}$

9.  $A$  - We must have  $c \sin \frac{\pi}{2} + 1 = 4 \frac{\pi}{2} + 2 \rightarrow c = 2\pi + 1$   $B = 2$   $C = f'(3) = e^3$  where  $f(x) = e^x$ .  $A + B + \ln C = \boxed{2\pi + 6}$

10.  $A = lcm \left\{ \frac{2\pi}{1/24}, \frac{2\pi}{1/12} \right\} = lcm \{48\pi, 24\pi\} = 48\pi$ .  $B = 500000$  Integrating  $d(t)$  over one period will make the contributions from  $\sin$  and  $\cos$  both zero, so the average value is

just  $\frac{1}{48\pi} \int_0^{48\pi} 500000 dx = 500000$ .  $C = 0$  Since the distance will not have changed, the average rate of change will be zero.  $A+B+C = \boxed{500000 + 48\pi}$

11. If the distance from the base to the wall is  $x$  and the distance from the floor to the top of the ladder is  $y$ , then  $x^2 + y^2 = 20^2$ , so implicit differentiation with respect to  $t$  yields

$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$ . Solving for  $\frac{dy}{dt}$  and plugging in  $x = 10$ ,  $\frac{dx}{dt} = 1$ ,  $y = \sqrt{20^2 - 10^2}$ , we

get  $\left| \frac{dy}{dt} \right| = \left| \frac{-10 \cdot 1}{\sqrt{300}} \right| = \boxed{\frac{\sqrt{3}}{3}}$ .

12.  $f'(x) = 6x^2 - 10x - 4 = 2(3x+1)(x-2) \rightarrow$  We must check  $x = -10, -1/3, 0, 2, 10$

$$f(-4) = -189 \quad f(-1/3) = 3 \frac{19}{27} \quad f(0) = 3 \quad f(2) = -9 \quad f(4) = 35$$

$A = \frac{100}{27} \quad B = -9 \quad C = \frac{5}{6}$  by setting  $0 = f''(x) = 12x - 10$ .  $ABC = \boxed{-\frac{250}{9}}$ .

13.  $\ln h(x) = g(x) \ln f(x) \rightarrow \frac{h'(x)}{h(x)} = \frac{g(x)f'(x)}{f(x)} + g'(x) \ln f(x)$ , so

$$h'(x) = f(x)^{g(x)} \left[ \frac{g(x)f'(x)}{f(x)} + g'(x) \ln f(x) \right].$$

Plugging in  $x = 1$  and using the values in

the question, we have  $h'(1) = 2^1 \left[ \frac{1 \cdot 3}{2} + 4 \cdot \ln 2 \right] = \boxed{3 + 8 \ln 2}$ .

14.  $\int_0^{24} \sqrt{1 + [f'(x)]^2} dx = \int_0^{24} \sqrt{1 + x} dx = \int_1^{25} \sqrt{u} du = \frac{2}{3} (25^{3/2} - 1^{3/2}) = \frac{2}{3} (124) = \boxed{\frac{248}{3}}$

15.  $A - f'(x) = -2xe^{-x^2} \rightarrow f''(x) = -2e^{-x^2} + 4x^2e^{-x^2} = e^{-x^2}(4x^2 - 2)$ , so  $f'$  attains its maximum at  $x = -\frac{\sqrt{2}}{2}$ , where its value is  $\sqrt{2}e^{-1/2}$ .  $B = 1$  since the Mean Value Theorem

guarantees for some  $c$  that  $\left| \frac{f(x) - f(y)}{x - y} \right| = f'(c) \leq \frac{\sqrt{2}}{\sqrt{e}} < 1$ .  $C = 1$  It's pretty easy to see

that the graph of  $f$  will intersect  $y = x$  at one spot.  $D = 0$  By above there is some  $c < 1$  where  $|x_{n+1} - x_n| = |f(x_n) - f(x_{n-1})| \leq c|x_n - x_{n-1}|$ , so by repeated application,

$|x_{n+1} - x_n| \leq c^{n-1}|x_2 - x_1|$  yielding  $\lim_{n \rightarrow \infty} |x_{n+1} - x_n| = 0$  since  $c^{n-1} \rightarrow 0$ . But this says the

difference between  $f(x_n)$  and  $x_n$  approaches zero, so they must have the same limit  $x$ .

Then  $x = \lim_{n \rightarrow \infty} x_n = \lim_{n \rightarrow \infty} f(x_n) = f(\lim_{n \rightarrow \infty} x_n) = f(x)$ , so  $x$  is a fixed point of  $f$ . It is pretty easy

to guess the fixed point of  $f$  by checking small numbers, and we find that  $f(0) = 0$ .

$A^2 + B + C + D = \boxed{\frac{2}{e} + 2}$ .