

Round 1 Alpha Region Bowl
2008 FAMAT State Convention

Consider the general equation $(2x + y - 4)(4x - y + 2) = k$.

1. What value(s) of k will yield a degenerate conic section?
2. When not a degenerate conic section, what type of conic does the equation represent?
3. What is the center of the conic when $k = 2008$?
4. Will any other value of k besides those in Parts A and C yield a different location for the center of the conic than that in Part C? (Yes or No)

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Round 2 Alpha Region Bowl
2008 FAMAT State Convention

1. Suppose all the zeros of the polynomial $f(x)$ are integers. If the leading coefficient and constant term of $f(x)$ are 1 and $20!$, respectively, what is the smallest positive integer that cannot be a root of $f(x)$?
2. What is the maximum number of possible intersections of a polynomial of degree 2008 and a polynomial of degree 5?
3. Suppose $f(x)$ is a polynomial and $q(x)$, $r(x)$ are the quotient and remainder, respectively, when $f(x)$ is divided by $(x-2)(x-1)$. If $f(2)=3$ and $f(1)=-1$, find $r(x)$. (Write your answer in the form $ax+b$.)
4. A polynomial with real coefficients has the property that if z is one of its roots, then the complex conjugate of z , \bar{z} , is also a root. If the coefficients are instead assumed to be complex (not necessarily real), does this property still hold? (Yes or No)

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Round 3 Alpha Region Bowl
2008 FAMAT State Convention

1. Find the sum of the entries of the matrix: $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{2008}$

2. Note $\begin{pmatrix} a & -b \\ b & a \end{pmatrix} \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} ac - bd \\ ad + bc \end{pmatrix}$. What is $\begin{pmatrix} \cos(5^\circ) & -\sin(5^\circ) \\ \sin(5^\circ) & \cos(5^\circ) \end{pmatrix}^{18} \begin{pmatrix} 2 \\ 3 \end{pmatrix}$?

3. Find the value of the following determinant:

$$\begin{vmatrix} 2 & 5 & -3 & 1 \\ 0 & -1 & 4 & 0 \\ 1 & 3 & 0 & 2 \\ 0 & 2 & -5 & 3 \end{vmatrix}$$

4. (Yes or No) Does there exist any ordered quadruplet (a, b, c, d) such that the following system of equations has no solution?:

$$\begin{aligned} 2w + 5x - 3y + z &= a \\ -x + 4y &= b \\ w + 3x + 2z &= c \\ 2x - 5y + 3z &= d \end{aligned}$$

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Round 4 Alpha Region Bowl

2008 FAMAT State Convention

1. Find the exact value of $\cos^2(22.5^\circ)$.
2. Find the maximum value of $\sin x + \cos x$.
3. Triangle ABC is inscribed in a circle of radius 4. If $BC = 7$, what is $\sin A$?
4. For the triangle in Part C, what is $\frac{b}{\sin B}$?

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Round 5 **Alpha Region Bowl**
2008 FAMAT State Convention

1. Find the following sum: $\sum_{i=0}^{\infty} \sin\left(\frac{\pi}{4}\right)\left(1 - \cos\left(\frac{\pi}{4}\right)\right)^i$.
2. If $\binom{n}{k}$ represents the standard binomial coefficient $\frac{n!}{k!(n-k)!}$, find the following sum: $\sum_{k=0}^5 \binom{11}{2k+1}$.
3. Find $i + i^2 + i^3 + \dots + i^{2007} + i^{2008}$.
4. Find $\sin 1^\circ + \sin 2^\circ + \sin 3^\circ + \dots + \sin 359^\circ + \sin 360^\circ$.

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Round 6 Alpha Region Bowl
2008 FAMAT State Convention

Consider the expression $(x - y - z)^8$. If the expression is expanded and like terms are collected, find each of the following:

1. The coefficient on the x^3yz^4 term
2. The number of terms
3. The sum of the coefficients on all the terms
4. The sum of the coefficients on all the terms containing a positive power of x

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Round 7 Alpha Region Bowl
2008 FAMAT State Convention

1. Find the exact value of $\sin\left(2\cos^{-1}\left(-\frac{3}{5}\right)\right)$.
2. At what value of x in the interval $[0, 2\pi)$ does the graph $y = \tan\left(\frac{1}{2}x - \frac{\pi}{6}\right)$ have a vertical asymptote?
3. Find the area of triangle ABC if $AB = 4$, $BC = 8$, and $m\angle B = 75^\circ$.
4. A triangle has sides of 2, 5, and $\sqrt{39}$. Find the measure of the largest angle in degrees.

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Round 8 Alpha Region Bowl
2008 FAMAT State Convention

1. Find the nearest integer to $\log_{16} 2008$
2. Find the sum of the distinct solutions for x over the reals: $x^{x^2-3x+2} - 1 = 0$
3. Let $c(x) = \frac{e^x + e^{-x}}{2}$ and $s(x) = \frac{e^x - e^{-x}}{2}$. Find $c^2(2008) - s^2(2008)$.
4. Solve for x over the reals in the following equation: $1 = \frac{e^x}{1 + 2e^{-x}}$

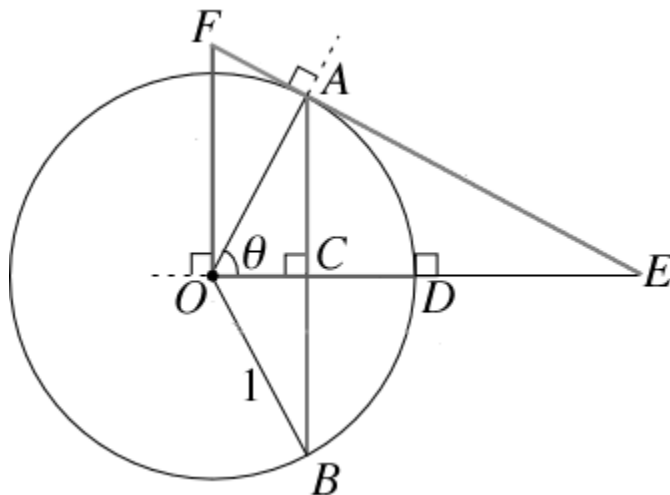
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Round 9 Alpha Region Bowl
2008 FAMAT State Convention

In the figure to the right, label each of the following using the correct trigonometric function $\sin \theta$, $\cos \theta$, $\tan \theta$, $\cot \theta$, $\sec \theta$ or $\csc \theta$:

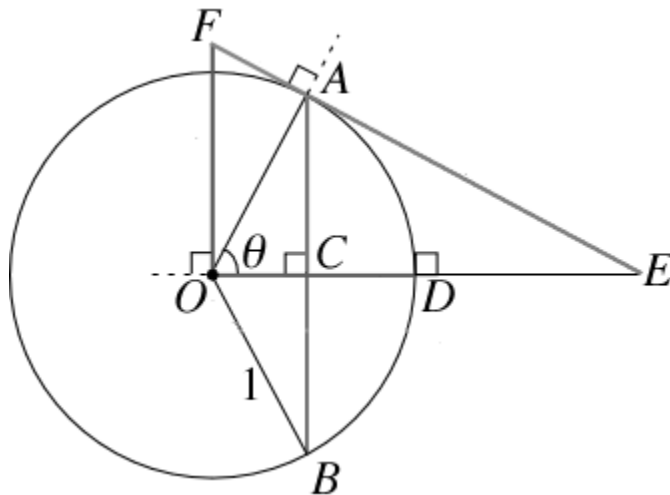
1. Length AE
2. Length OE
3. Length FA
4. Length OF



Round 9 Alpha Region Bowl
2008 FAMAT State Convention

In the figure to the right, label each of the following using the correct trigonometric function $\sin \theta$, $\cos \theta$, $\tan \theta$, $\cot \theta$, $\sec \theta$ or $\csc \theta$:

1. Length AE
2. Length OE
3. Length FA
4. Length OF



Round 10 Alpha Region Bowl
2008 FAMAT State Convention

1. Find the probability of a coin landing heads up exactly 3 times out of 5 flips
2. If x and y are chosen completely at random from the interval $[0,4]$, find the probability that $x + y \geq 6$.
3. The number of possible committees consisting of 9 people chosen from a fixed group of 20 people is 167,960. The number of possible committees consisting of 10 people chosen from a fixed group of 21 people is 352,716. How many possible committees consisting of 10 people chosen from a fixed group of 20 people are there?
4. A family has two children. Find the probability that one is a girl, given that one is a boy.

Round 10 Alpha Region Bowl
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2. If x and y are chosen completely at random from the interval $[0,4]$, find the probability that $x + y \geq 6$.
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