#1

A. $\boxed{0}$ (We get the equation of two lines)
B. $\boxed{\text{Hyperbola}}$ (the discriminant is positive)
C. Intuitively, it will be where the lines $2x+y-4=0$ and $4x-y+2=0$ meet. Solving for *y* in one equation and replacing, we find $x=\frac{1}{3}$, and thus $\boxed{\left(\frac{1}{3},\frac{10}{3}\right)}$ is the center.
D. $\boxed{\text{No}}$. Changing *k* will only change the constant term of the equation, which has no effect on where the center is located.

#2

A. The constant term is plus or minus the product of the roots, and the smallest positive integer which does not evenly divide 20! is $\boxed{23}$.
B. The difference of the two polynomials will be a polynomial of degree 2008, which has at most 2008 zeros…hence, the two intersect at most $\boxed{2008}$ times.
C. $f(x)=q(x)(x-2)(x-1)+r(x)$, so $3=f(2)=r(2),\ -1=f(1)=r(1)$. Now solve the resulting system of equations: $2a+b=3,\ a+b=-1 \rightarrow a=4, b=-5$. $\boxed{4x-5}$
D. $\boxed{\text{No}}$: Simply take then polynomial $x-i=0$ …the only root is *i*.

#3

A. Finding the first couple powers, it is easy to guess (and prove with induction) that $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}^n=\begin{bmatrix}1 & n\\0 & 1\end{bmatrix}$…The sum of the entries of $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}^{2008}=\begin{bmatrix}1 & 2008\\0 & 1\end{bmatrix}$ is just $\boxed{2010}$.
B. Since $(a+bi)(c+di)=(ac-bd)+(ad+bc)i$, the note shows that $\begin{pmatrix}\cos(5°) & -\sin(5°)\\ \sin(5°) & \cos(5°)\end{pmatrix}^{18}\begin{pmatrix}2\\3\end{pmatrix}=\begin{pmatrix}x\\y\end{pmatrix}$, where *x* is the real part and *y* is the imaginary part of $(\cos 5°+i\sin 5°)^{18}(2+3i)=(\cos 90°+i\sin 90°)(2+3i)=-3+2i \rightarrow \boxed{\begin{pmatrix}-3\\2\end{pmatrix}}$
C. Add -2 times row 3 to row 1 to get all zeros in the first column except for in row 3. Now expand along the first column:

$$\begin{vmatrix}0 & -1 & -3 & -3\\0 & -1 & 4 & 0\\1 & 3 & 0 & 2\\0 & 2 & -5 & 3\end{vmatrix}=+1\begin{vmatrix}-1 & -3 & -3\\-1 & 4 & 0\\2 & -5 & 3\end{vmatrix}=\boxed{-12}.$$

D. Because the determinant in part C was non-zero, Cramer's Rule says there is a solution no matter what choice of *a*, *b*, *c* or *d* are used. $\boxed{\text{No}}$.

#4

A. $\cos 45° = 2\cos^2 22.5° - 1 \rightarrow \cos^2 22.5° = \boxed{\frac{2+\sqrt{2}}{4}}$ or $\boxed{\frac{1}{2}+\frac{\sqrt{2}}{4}}$

B. $\sin x + \cos x = \frac{1}{\sqrt{2}/2}\left(\frac{\sqrt{2}}{2}\sin x + \frac{\sqrt{2}}{2}\cos x\right) = \frac{1}{\sqrt{2}/2}\sin(x+\frac{\pi}{4})$, so the maximum value is $\boxed{\frac{1}{\sqrt{2}/2} = \sqrt{2}}$.

C. For a general triangle ABC, $\frac{a}{\sin A} = 2r$, where r is the radius of the circumscribed circle. (See a derivation of the Law of Sines for a short proof.) Then, for our triangle, $\boxed{\sin A = \frac{7}{8}}$.

D. Using the Law of Sines, the ratio must be $\boxed{\frac{7}{7/8} = 8}$.

#5

A. $\frac{\sin\frac{\pi}{4}}{1-\left(1-\cos\frac{\pi}{4}\right)} = \tan\frac{\pi}{4} = \boxed{1}$

B. The sum of the coefficients in the 11th row of Pascal's Triangle is 2^{11} …the sum of every other coefficient is half that, or $\boxed{2^{10} = 1024}$.

C. Four consecutive powers of i add to 0, and since 4 divides 2008, the answer is $\boxed{0}$.

D. $\sin(x) = -\sin(360° - x)$, so we can pair up and cancel all the terms except $\sin(180°)$ and $\sin(360°)$, but these are both 0. The answer is then $\boxed{0}$.

#6

A. $\boxed{\frac{8!}{3!1!4!} = 280}$

B. Number of ways of distributing 8 identical items in 3 cells → $\boxed{\frac{(8+3-1)!}{8!2!} = 45}$

C. Plug in $x = y = z = 1$ → $(1-1-1)^8 = \boxed{1}$

D. Plug in 0 for x and 1 for y, z to get the sum of the coefficients on all terms which do *not* contain a positive power of x. Subtract this from the sum of all the coefficients → $(1-1-1)^8 - (0-1-1)^8 = 1 - 256 = \boxed{-225}$

#7

A. $\sin\left(2\cos^{-1}\left(-\frac{3}{5}\right)\right) = 2\sin\left(\cos^{-1}\left(-\frac{3}{5}\right)\right)\cos\left(\cos^{-1}\left(-\frac{3}{5}\right)\right) = 2\left(\frac{4}{5}\right)\left(-\frac{3}{5}\right) = \boxed{-\frac{24}{25}}$

B. $\tan x$ has an asymptote at $x = \frac{\pi}{2}$, so $\tan\left(\frac{1}{2}x - \frac{\pi}{6}\right)$ will have an asymptote when $\frac{1}{2}x - \frac{\pi}{6} = \frac{\pi}{2}$, or $\boxed{x = \frac{4\pi}{3}}$. (This is the only solution since the period of y is 2π.)

C. $\frac{1}{2}(4)(8)\sin(75°) = 16\sin(30° + 45°) = 16(\sin 30°\cos 45° + \sin 45°\cos 30°)$
$= 16\left(\frac{1}{2}\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2}\right) = \boxed{4\sqrt{2} + 4\sqrt{6}}$ or $\boxed{4\left(\sqrt{2} + \sqrt{6}\right)}$ or $\boxed{4\sqrt{2}\left(1 + \sqrt{3}\right)}$

D. The largest angle, A, is opposite the largest side, which is the side of length $\sqrt{39}$.
By the law of cosines, $\cos A = -\frac{\left(\sqrt{39}\right)^2 - 2^2 - 5^2}{2(2)(5)} = -\frac{1}{2}$, so $A = \boxed{120°}$.

#8

A. $16 = 2^4$ and $2^{10} = 1024 < 2008 < 2048 = 2^{11}$, so
$2.5 = \log_{16} 1024 < \log_{16} 2008 < \log_{16} 2048 = 2.75 \rightarrow \boxed{3}$

B. $x^{x^2-3x+2} - 1 = 0 \rightarrow x = 1$ or -1 or $x^2 - 3x + 2 = 0 \rightarrow x = 1, -1, 2 \rightarrow \boxed{2}$

C. $\left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2 = \frac{1}{4}\left(e^{2x} + 2 + e^{-2x} - e^{2x} + 2 - e^{-2x}\right) = \boxed{1}$.

D. $1 = \frac{e^x}{1 + 2e^{-x}} \rightarrow 1 + 2e^{-x} = e^x \rightarrow e^{2x} - e^x - 2 = 0$, so $\left(e^x - 2\right)\left(e^x + 1\right) = 0 \rightarrow e^x = 2$,
and $\boxed{x = \ln 2}$

#9

A. $AE = \frac{AE}{AO} = \frac{AC}{CO} = \boxed{\tan\theta}$ (note the similar triangles AEO and ACO)

B. From part A, $\sin\theta = \frac{AE}{OE} = \frac{\tan\theta}{OE} \rightarrow OE = \frac{1}{\cos\theta} = \boxed{\sec\theta}$

C. $\boxed{\cot\theta}$ (same method as in part A except the angle θ has moved)

D. $\boxed{\csc\theta}$ (use method in part B)

#10

A. Binomial distribution: $\boxed{\binom{5}{3}\left(\frac{1}{2}\right)^5 = \frac{5}{16}}$

B. Graph the square region from which the numbers are chosen and shade in the part which satisfies the inequality (the points in the square above the line $y = 6 - x$). The area of the shaded region divided by the area of the square is our probability. The area of the shaded region is just that of a triangle with area $\frac{1}{2} \cdot 2 \cdot 2 = 2$, and the square has area 16 → $\boxed{\frac{1}{8}}$.

C. Because $\binom{20}{9} + \binom{20}{10} = \binom{21}{10}$ (an entry in Pascal's Triangle is the sum of the two entries above it) we can simply subtract: $\boxed{352,716 - 167,960 = 184,756}$.

D. $\boxed{2/3}$ For a couple with 2 children, there are 4 possible equally likely combinations: BB, BG, GB, GG…since at least one is a boy, there are now 3 equally likely combinations BB, BG, GB, of which 2 contain a girl.