

Alpha Trigonometry – FAMAT State Convention 2008

For all questions, answer E. NOTA means none of the above answers are correct.

- Let  $\cos(4x) = A\cos^4(x) + B\cos^3(x) + D\cos^2(x) + E\cos(x) + F$ . Compute  $A+B+D+E+F$ .
  - 1
  - 0
  - 1
  - 2
  - NOTA
- Given  $y = 0.75\sin(2x - 1)$ ; Let A = the magnitude of the phase shift of y; B = the amplitude of y and C = the period of y. Find  $A^2 - B + C \div \pi$ .
  - 1
  - 0.5
  - 0
  - 0.5
  - NOTA
- Find:  $\lim_{x \rightarrow \infty} \left( \frac{x+1-\cos^2(x)}{2x^2} \right)$ 
  - 1
  - 0
  - 1
  - Does not exist
  - NOTA
- What is the acute angle of rotation (in degrees) needed to eliminate the xy-term from the equation:  
 $7x^2 + \sqrt{3}xy + 6y^2 - 4x + 5y - 8 = 0$ ?
  - 15
  - 30
  - 45
  - 60
  - NOTA
- Which one of the following is one of the fifth roots of 1? Note: Answers are in degrees.
  - $\cos(216) + i\sin(216)$
  - $\cos(192) + i\sin(192)$
  - $\cos(108) + i\sin(108)$
  - $\cos(60) + i\sin(60)$
  - NOTA
- Let A= the absolute value of b such that  $0.5r = 5 + b\cos(\theta)$  is a cardioid. Let B = the number of leaves in the polar curve  $r = 2\sin(3\theta)$ . Find: A - B.
  - 1
  - 3
  - 4
  - 7
  - NOTA
- What is the equivalent of  $\pi/32$  radians?
  - $5^\circ 6' 25''$
  - $5^\circ 15' 12''$
  - $5^\circ 37' 30''$
  - $5^\circ 45' 45''$
  - NOTA
- Find the rectangular coordinates of the polar point  $(-3, 150^\circ)$ .
  - $(\frac{3\sqrt{3}}{2}, -\frac{3}{2})$
  - $(-\frac{3}{2}, \frac{1}{2})$
  - $(\frac{3}{2}, 3\sqrt{3})$
  - $(3\sqrt{3}, -\frac{3}{2})$
  - NOTA
- In  $\triangle DEF$ ,  $d=6$ ,  $D=30^\circ$  and  $F=45^\circ$ . Find f.
  - $6\sqrt{2}$
  - $3\sqrt{2}$
  - $2\sqrt{2}$
  - $\sqrt{2}$
  - NOTA
- In which quadrant is the terminal side of the angle  $2008^\circ$ ?
  - I
  - II
  - III
  - IV
  - NOTA

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11. Determine:  $1 + \sin(x) + \sin^2(x) + \sin^3(x) + \sin^4(x) + \dots$  where  $x = 45^\circ$ .
- $4 + \sqrt{2}$
  - $2 + \sqrt{2}$
  - $2 + 2\sqrt{2}$
  - $4 + 2\sqrt{2}$
  - NOTA
12. Given  $\triangle ABC$  with  $a=5$ ,  $b=12$ ,  $c=13$  and angle  $A = \Theta$ . Find  $\csc(2\Theta) - \cot(\Theta)$
- $-\frac{119}{120}$
  - $-\frac{67}{169}$
  - $-\frac{37}{60}$
  - $-\frac{137}{156}$
  - NOTA
13. A cosine curve has a frequency of  $1/3$ , which of the following could be the curve:
- $y = 3\cos(x - 3)$
  - $y = 2\cos(\frac{1}{3}x + 4)$
  - $y = \frac{1}{3}\cos(2x - 4)$
- I only
  - III only
  - I and II only
  - II and III only
  - NOTA
14. In  $\triangle PQR$ ,  $p=8$ ,  $q=9$  and  $R=60^\circ$ . Find the area of  $\triangle PQR$ .
- $18\sqrt{3}$
  - $9\sqrt{3}$
  - $18\sqrt{2}$
  - $9\sqrt{2}$
  - NOTA
15. In  $\triangle XYZ$ ,  $x=4$ ,  $y=5$  and  $Z=60^\circ$ . Find  $z$ .
- $2\sqrt{3}$
  - $\sqrt{15}$
  - $3\sqrt{2}$
  - $\sqrt{21}$
  - NOTA
16. Simplify:  $\sin(x + 45^\circ) - \cos(x + 30^\circ)$  into the form  $a*\sin(x) + b*\cos(x)$ . Find  $a - b$ .
- $\frac{1 - \sqrt{3}}{2}$
  - $\frac{-1 + \sqrt{3}}{2}$
  - $\frac{1 + \sqrt{3}}{2}$
  - $\frac{-1 - \sqrt{3}}{2}$
  - NOTA
17. Find the roots for  $y = \cos^2(x) + \frac{1}{2}\sin(x) - \frac{1}{2}$  on the interval  $(0, 360^\circ)$ .
- $\{210^\circ, 330^\circ\}$
  - $\{90^\circ, 210^\circ, 330^\circ\}$
  - $\{90^\circ, 240^\circ, 300^\circ\}$
  - $\{90^\circ, 210^\circ, 270^\circ, 330^\circ\}$
  - NOTA
18. Find the Cartesian equation for the polar graph:  $r = \frac{3}{4 - \cos(\Theta)}$ .
- $16x^2 + 15y^2 - 6y - 9 = 0$
  - $15x^2 + 16y^2 - 6x - 9 = 0$
  - $16x^2 + 9y^2 - 4x - 3y - 12 = 0$
  - $9x^2 + 16y^2 - 3x - 4y - 12 = 0$
  - NOTA
19. Which of the following angles is coterminal with  $\frac{275\pi}{3}$  radians?
- $30^\circ$
  - $120^\circ$
  - $240^\circ$
  - $300^\circ$
  - NOTA

20. For the graph  $y = 3\sin(x) - 6\cos(x)$ . Let  $A =$  the amplitude of  $y$ , let  $B =$  the period of the graph of  $y$  in degrees. Find  $B \div A^2$ .
- A. 8  
B. 4  
C. 2  
D. 1  
E. NOTA
21. Given  $\sin(x) = \frac{2}{7}$ , find  $\tan(2x)$ . Assume the terminal side of  $x$  lies in the first quadrant.
- A.  $\frac{6\sqrt{3}}{41}$   
B.  $\frac{9\sqrt{5}}{41}$   
C.  $\frac{12\sqrt{5}}{41}$   
D.  $\frac{18\sqrt{3}}{41}$   
E. NOTA
22. Find the polar coordinates equivalent to the Cartesian coordinates  $(-5, -5\sqrt{3})$ .
- A.  $(10, 210^\circ)$   
B.  $(5, 240^\circ)$   
C.  $(10, 240^\circ)$   
D.  $(5, 210^\circ)$   
E. NOTA
23. Simplify:  $\left(\frac{2\text{cis}(48)}{-3\text{cis}(66)}\right)^5$ .
- A.  $\frac{32}{729}i$   
B.  $\frac{8}{81}i$   
C.  $\frac{16}{81}i$   
D.  $\frac{32}{243}i$   
E. NOTA
24. Myron is a discus thrower. He is able to spin his body around at a speed of 2 revolutions per second. Given he spins around with the discus in his hand (and his arm is 3 feet long) – at what speed does the discus leave his hand? Assume the discus travels in a circular path before being released and neglect the width of Myron's body. Give your answer in yards per minute.
- A.  $4\pi$   
B.  $12\pi$   
C.  $72\pi$   
D.  $240\pi$   
E. NOTA
25. Suppose you select a number at random on the interval  $(-360, 360)$  and that number represents a degree measure. What is the probability that you select a number where the sine of its angle, measured in degrees, is less than  $-0.5$ ?
- A.  $1/2$   
B.  $5/12$   
C.  $1/3$   
D.  $1/4$   
E. NOTA
26. If  $\cos(x) = a/b$ , what is  $\tan(x)$ ? Assume  $a \neq b \neq 0$ .
- A.  $\frac{\sqrt{b^2 - a^2}}{a}$   
B.  $\frac{\sqrt{b^2 - a^2}}{b}$   
C.  $\frac{a}{\sqrt{b^2 - a^2}}$   
D.  $\frac{b}{\sqrt{b^2 - a^2}}$   
E. NOTA

27. Using the half-angle formula, find  $\sin(22.5^\circ)$ .

- A.  $\sqrt{\frac{1}{2} - \sqrt{2}}$
- B.  $\frac{1}{2}\sqrt{2 - \sqrt{2}}$
- C.  $\frac{1}{2}\sqrt{4 - \sqrt{2}}$
- D.  $\frac{1}{2}\sqrt{5 - \sqrt{3}}$
- E. NOTA

28. What is the range of the cosecant function?

- A.  $(-\infty, -1] \cup [1, \infty)$
- B.  $(-\infty, -1) \cup (1, \infty)$
- C.  $(-1, 1)$
- D.  $[-1, 1]$
- E. NOTA

29. Given  $\sinh(x) = \frac{e^x - e^{-x}}{2}$ , what is  $\operatorname{csch}(\ln 9)$ ?

- A.  $42/9$
- B.  $9/42$
- C.  $9/40$
- D.  $40/9$
- E. NOTA

30. Compute the cosine of the acute angle formed by  $\langle -5, 3, 0 \rangle$  and  $\langle 1, 5, 7 \rangle$ .

- A.  $\frac{\sqrt{51}}{37}$
- B.  $\frac{\sqrt{102}}{51}$
- C.  $\frac{\sqrt{73}}{21}$
- D.  $\frac{\sqrt{133}}{46}$
- E. NOTA