For all questions, E. NOTA means none of the above answers is correct.

1. Evaluate: $\begin{bmatrix} -2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} 1 & -3 \\ 4 & 1 \end{bmatrix}$.

A. $\begin{bmatrix} -20 & -2 \\ -14 & 9 \end{bmatrix}$ B. $\begin{bmatrix} -2 & -18 \\ 4 & 5 \end{bmatrix}$ C. $\begin{bmatrix} -1 & 3 \\ 5 & 6 \end{bmatrix}$ D. $\begin{bmatrix} 22 & 12 \\ 21 & 2 \end{bmatrix}$ E. NOTA

2. Evaluate: $\begin{bmatrix} -2 & 6 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 4 & 1 \end{bmatrix}$.

A. $\begin{bmatrix} -20 & -2 \\ -14 & 9 \end{bmatrix}$ B. $\begin{bmatrix} -2 & -18 \\ 4 & 5 \end{bmatrix}$ C. $\begin{bmatrix} -1 & 3 \\ 5 & 6 \end{bmatrix}$ D. $\begin{bmatrix} 22 & 12 \\ 21 & 2 \end{bmatrix}$ E. NOTA

3. Evaluate: $\langle -1,4,2\rangle \cdot \langle 0,5,3\rangle$

A. -9 B. 6 C. 19 D. 26 E. NOTA

4. Evaluate: $\left\| \langle -1,4,2\rangle \times \langle 0,5,3\rangle \right\|$

A. $\sqrt{26}$ B. $\sqrt{38}$ C. $\sqrt{107}$ D. $\sqrt{714}$ E. NOTA

5. Let A_n be the $n \times n$ matrix such that the a_{ij} element of A_n is $i + j$. What is $|A_{2008}|$?

A. 0 B. 2008 C. 2008! D. 2008^{2008} E. NOTA

6. What is the tangent of the acute angle between $\mathbf{i} + \mathbf{j}$ and $\mathbf{j} + \mathbf{k}$?

A. 0 B. $\frac{\sqrt{3}}{3}$ C. 1 D. $\sqrt{3}$ E. NOTA

7. Find the inverse of $\begin{bmatrix} 4 & -3 \\ 5 & -4 \end{bmatrix}$.

A. $\begin{bmatrix} 4 & -3 \\ 5 & -4 \end{bmatrix}$ B. $\begin{bmatrix} 4 & -5 \\ 3 & -4 \end{bmatrix}$ C. $\begin{bmatrix} -4 & 3 \\ -5 & 4 \end{bmatrix}$ D. $\begin{bmatrix} -4 & 5 \\ -3 & 4 \end{bmatrix}$ E. NOTA

8. Which of the following vectors is NOT a unit vector?

A. $\frac{9\sqrt{154}}{154}\mathbf{i} + \frac{8\sqrt{154}}{154}\mathbf{j} - \frac{3\sqrt{154}}{154}\mathbf{k}$ B. $\frac{3\sqrt{178}}{178}\mathbf{i} - \frac{5\sqrt{178}}{178}\mathbf{j} + \frac{12\sqrt{178}}{178}\mathbf{k}$

C. $\frac{11\sqrt{291}}{291}\mathbf{i} + \frac{\sqrt{291}}{291}\mathbf{j} + \frac{13\sqrt{291}}{291}\mathbf{k}$ D. $-\frac{7\sqrt{554}}{554}\mathbf{i} - \frac{8\sqrt{554}}{554}\mathbf{j} + \frac{21\sqrt{554}}{554}\mathbf{k}$ E. NOTA

9. Use a determinant to find the area of the triangle whose vertices are $(1,0)$, $(3,5)$, and $(2,1)$.

A. $\frac{3}{4}$ B. $\frac{3}{2}$ C. 3 D. 6 E. NOTA

10. A square matrix A is said to be idempotent if $A^2 = A$. If $\begin{bmatrix} x & 2x \\ 2x & y \end{bmatrix}$ is idempotent, then what is the maximum value of $x+y$?

A. 0 B. 1 C. 2 D. 3 E. NOTA

11. Evaluate: $\begin{vmatrix} 1 & 0 & 2 & 4 \\ 0 & -1 & 3 & 0 \\ 0 & 2 & 5 & -3 \\ 1 & 0 & 0 & 2 \end{vmatrix}$

A. -72 B. -24 C. 7 D. 28 E. NOTA

12. If $A = \begin{bmatrix} 1 & 17 & 3 \\ x & -8 & y \\ 3 & -5 & -11 \end{bmatrix}$ is symmetric, then what is $x+y$?

A. -18 B. 12 C. 23 D. 39 E. NOTA

13. Use Cramer's rule to find x if $x+y+z=2$, $z-x=-3$, and $2x-y+3z=7$.

A. -1 B. $-\frac{3}{7}$ C. 0 D. 3 E. NOTA

14. Let **a**, **b**, and **c** be three dimensional vectors. Which of the following is NOT true?

A. $\mathbf{a}\times\mathbf{b} = -\mathbf{b}\times\mathbf{a}$ B. $\mathbf{a}\times(\mathbf{b}+\mathbf{c}) = (\mathbf{a}\times\mathbf{b})+(\mathbf{a}\times\mathbf{c})$

C. $(\mathbf{a}\times\mathbf{b})\cdot\mathbf{c} = \mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})$ D. $(\mathbf{a}\times\mathbf{b})\times\mathbf{c} = \mathbf{a}\times(\mathbf{b}\times\mathbf{c})$ E. NOTA

For questions 15, 16, and 17, let $A = \begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & 1 & 1 \\ 3 & 5 & 7 & 10 \end{bmatrix}$.

15. What is the rank of A?

A. 1 B. 2 C. 3 D. 4 E. NOTA

16. What is the trace of A?

A. 7 B. 10 C. 13 D. 16 E. NOTA

17. What is the sum of the a_{13} and a_{32} elements of A?

A. -1 B. 1 C. 3 D. 5 E. NOTA

18. Let A be a 3×7 matrix and B be a 5×7 matrix. Which of the following matrix products is defined?

A. AB B. BA C. AB^{T} D. $B^{\mathrm{T}}A$ E. NOTA

19. What is the sum of the values of x such that $\begin{bmatrix} x & 1 & x \\ -2 & x & 3 \\ x & 0 & 7 \end{bmatrix}$ is singular?

A. 7 B. 0 C. -3 D. -14 E. NOTA

20. Let $\mathbf{a}$ and $\mathbf{b}$ be nonzero three dimensional vectors. Which of the following is equal to $\|\mathbf{a}\|^2\|\mathbf{b}\|^2$?

A. $\dfrac{\|\mathbf{a}\times\mathbf{b}\|^2}{(\mathbf{a}\cdot\mathbf{b})^2}$ B. $\|\mathbf{a}\times\mathbf{b}\|^2\cdot(\mathbf{a}\cdot\mathbf{b})^2$ C. $\|\mathbf{a}\times\mathbf{b}\|^2+(\mathbf{a}\cdot\mathbf{b})^2$ D. $\|\mathbf{a}\times\mathbf{b}\|^2-(\mathbf{a}\cdot\mathbf{b})^2$ E. NOTA

21. Suppose $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\lambda\begin{bmatrix} x \\ y \end{bmatrix}$ where λ is a real constant and $\begin{bmatrix} x \\ y \end{bmatrix}\neq\begin{bmatrix} 0 \\ 0 \end{bmatrix}$. What is the value of $\lambda+y$?

A. -1 B. 0 C. 1 D. 2 E. NOTA

22. Evaluate: $\begin{bmatrix} 1 & 0 & 1 \\ 3 & -1 & 1 \\ 0 & 1 & 1 \end{bmatrix}^{-1}$

A. $\begin{bmatrix} 1 & 3 & 0 \\ 0 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ B. $\begin{bmatrix} 1 & -3 & 0 \\ -2 & 2 & 1 \\ -1 & 3 & 1 \end{bmatrix}$ C. $\begin{bmatrix} -2 & 1 & 1 \\ -3 & 1 & 2 \\ 3 & -1 & -1 \end{bmatrix}$

D. The inverse does not exist E. NOTA

23. Which of the following vectors is normal to the plane $2x-3y+z=7$?

A. $\mathbf{i}+\mathbf{j}-\mathbf{k}$ B. $2\mathbf{i}-3\mathbf{j}+\mathbf{k}$ C. $\mathbf{i}-3\mathbf{j}+2\mathbf{k}$ D. $5\mathbf{i}+10\mathbf{j}+6\mathbf{k}$ E. NOTA

24. Let $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ represent three adjacent edges of a parallelepiped. Which of the following gives the volume of the parallelepiped?

A. $|\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})|$ B. $\|\mathbf{a}\|\cdot\|\mathbf{b}\|\cdot\|\mathbf{c}\|$ C. $\|\mathbf{a}\times(\mathbf{b}\times\mathbf{c})\|$ D. $\dfrac{\|\mathbf{b}\times\mathbf{c}\|}{\|\mathbf{a}\|}$ E. NOTA

25. Let $A=\begin{bmatrix} 1 & -2 & 0 \\ 1 & 6 & -3 \end{bmatrix}$. What is $\det(A^{\mathrm{T}}A)$?

A. 251 B. 502 C. 1004 D. 2008 E. NOTA

26. Let $\mathbf{y} = \mathbf{x}\boldsymbol{\beta}$, where $\mathbf{y}$ is a 7×1 vector, $\mathbf{x}$ is a 7×5 matrix, and $\boldsymbol{\beta}$ is a 5×1 vector. If $\mathbf{x}$, $\mathbf{y}$, and $\boldsymbol{\beta}$ are all nonzero, and the rank of $\mathbf{x}$ is 5, then what is $\boldsymbol{\beta}$ expressed in terms of $\mathbf{x}$ and $\mathbf{y}$?

A. $\mathbf{x}^{-1}\mathbf{y}$ B. $\mathbf{y}\mathbf{x}^{-1}$ C. $\left(\mathbf{x}^{\mathrm{T}}\mathbf{x}\right)^{-1}\mathbf{x}^{\mathrm{T}}\mathbf{y}$ D. $\mathbf{x}^{\mathrm{T}}\mathbf{y}\left(\mathbf{x}^{\mathrm{T}}\mathbf{x}\right)^{-1}$ E. NOTA

27. The velocity of a particle at time t is given by $v(t) = \left(t^2 + 2t - 1\right)\mathbf{i} + \left(\sqrt{t} + 2\right)\mathbf{j} + e^{\sin(t)}\mathbf{k}$. What is the speed of the particle when $t = 0$?

A. 2 B. $\sqrt{6}$ C. $2\sqrt{2}$ D. $\sqrt{10}$ E. NOTA

28. Find the scalar projection of $5\mathbf{i} - 3\mathbf{j} - 5\mathbf{k}$ along $9\mathbf{i} + 4\mathbf{j} + \mathbf{k}$.

A. $\sqrt{2}$ B. $2\sqrt{2}$ C. $3\sqrt{2}$ D. 4 E. NOTA

29. Evaluate: $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}^{2008}$

A. $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ B. $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ C. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ D. $\begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$ E. NOTA

30. What is the magnitude of $2\mathbf{i} + 9\mathbf{j} - 6\mathbf{k}$?

A. 5 B. 11 C. 14 D. 22 E. NOTA