

1. **D.** The amount is going to be  $e^{0.015*3} = e^{0.045}$  since each year, the amount in the bank gets multiplied by  $e^{0.015}$
2. **B.** Note that  $K \sin(a+b) = K \sin a \cos b + K \cos a \sin b$ , so  
 $K \sin(a+b) + K \sin(a-b) = 2K \sin a \cos b$ , which is of the form we have, so just pick  $K = \frac{A}{2}$ ,  $ux = m+n = 7x+x$ ,  $vx = m-n = 7x-x \rightarrow K = 5, u = 8, v = 6$  so  
 $10 \sin(7x) \cos(x) = 5 \sin(8x) + 5 \sin(6x)$ , so the sum is 24.
3. **C.** Doubling the number of ducks doubles the number of eggs laid. Doubling the number of days doubles the number of eggs laid as well, so this will quadruple it to 36.
4. **B.** Note that  $\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}$ , so the sum of the reciprocals of 2 numbers is just the sum over the product, so ours is  $\frac{33}{270} = \frac{11}{90}$
5. **A.** His net trip was 30 miles east and 40 miles south, so my Pythagorean Theorem, the total distance from home is 50 miles.
6. **E.** 22800 is 4 half-lives, so  $1/16^{\text{th}}$  will remain, so 6.25 will remain.
7. **E.** All are equivalent and just different features of matrices with determinant 0.
8. **B.** There are 12 face cards, so there are 12 choose 3 = 220 ways to select 3 cards, and if one of them is the king of clubs, there are 11 choose 2 = 55 ways to pick the other 2 cards, so the probability of picking the king of clubs is  $55/220 = \frac{1}{4}$ .
9. **C.** The area of a parallelepiped box is the magnitude of the determinant of the vector space in a matrix, so it is  $\begin{vmatrix} 2 & 1 & 2 \\ 2 & -1 & 2 \\ 0 & 1 & 1 \end{vmatrix} = 4$ .
10. **C.** The system of three equations with three unknowns is  $x + y + z = 100$ ,  
 $4x + 2y + 3z = 300$  and  $100x + 20y + 50z = 5600 \rightarrow 10x + 2y + 5z = 560$ . To find  $y$ , use Cramer's Rule:  $y = \frac{\begin{vmatrix} 4 & 300 & 3 \\ 10 & 560 & 5 \\ 1 & 100 & 1 \end{vmatrix}}{\begin{vmatrix} 4 & 2 & 3 \\ 10 & 2 & 5 \\ 1 & 1 & 1 \end{vmatrix}} = \frac{60}{2} = 30$
11. **D.** The angle between the hands of the clock is  $\left| 30H - \frac{11}{2}M \right| = |60 - 253| = 193$ , so the smaller angle is  $167^\circ$ .

12. **A.** There are  $2^4 = 16$  ways they can run since each can either go forwards or backwards, but only 2 ways (all forwards or all backwards) will lead to no collisions, so the probability is  $\frac{1}{8}$ .
13. **D.** Volume of a cone is proportional to  $hr^2$ , so  $(1.2h)(.9r^2) = .968hr^2$ , so it is decreased by 3.2%
14. **A.** In order for it not to slip, the linear velocities of the two wheels must be the same, so say A's rotational velocity is  $a$  and B's is  $b$ .  $18\pi a = 12\pi b \rightarrow \frac{a}{b} = \frac{2}{3}$ .
15. **C.**  $a_n = b_{n-1} + a_{n-1}$  and  $b_n = a_{n-1} \rightarrow b_{n-1} = a_{n-2}$ , so  $a_n = a_{n-2} + a_{n-1}$ , which is the  $(n+1)^{\text{th}}$  Fibonacci number, so  $a_9 = 89$
16. **E.** Since there are only 4 types of coins, after the 5<sup>th</sup> draw, you are guaranteed to have at least one pair.
17. **D.** The total crust is  $16\pi$  long, so if you have 4 inches of crust, that is  $\frac{1}{4\pi}$  of the entire pizza, whose area is  $64\pi$ , so you have 16 square inches.
18. **B.** If I got more heads than tails, I either got 2 heads and 1 tail (3 ways to do so) or 3 heads and no tails (1 way to do so), so out of the 4 possible cases, only 1 is no tails.
19. **A.** Since the discriminant is  $5 > 0$ , it is a hyperbola.
20. **B.** The probability that the first person gets their dish correct is  $\frac{1}{8}$ , so the expected number of correct dishes served to the first person is  $\frac{1}{8}$ , and since we didn't make any assumption about picking which person is first, this can apply to all people, so by linearity of expectation, the expected number of total correct plates is 1.
21. **E.** We know that  $\cos \theta = \frac{u \cdot v}{\|u\| \|v\|} = \frac{-4}{\sqrt{6}\sqrt{12}} = \frac{-\sqrt{2}}{3}$ , so by the Pythagorean identity:  
 $\sin^2 \theta = \frac{7}{3}$ , and since we're looking for the smaller angle,  $\sin \theta > 0 \rightarrow \sin \theta = \frac{\sqrt{7}}{3}$ .
22. **C.** The x-coordinate of the vertex is  $-\frac{-2}{2} = 1$ , so the y-coordinate is  
 $1^2 - 2 + 6 = 5$ , so the sum is  $1 + 5 = 6$
23. **E.** The length of the diagonal is  $\sqrt{\sin^2 x + \cos^2 x + \tan^2 x} = \sqrt{1 + \tan^2 x} = \sec x$ , which has a minimum of 1 at  $x = 0$  and is strictly increasing in the first quadrant, so there is no minimum in the provided region.
24. **C.** The football follows the path  $y = h - \frac{h}{400}(x-20)^2$  so the maximum is at  
 $x = 20$ , 10 feet after it was kicked is at  $x = 10 \rightarrow y = \frac{3}{4}h$

25. **C.**  $t = j^2$ ,  $t + 8 = 2(j + 8) \rightarrow j^2 + 8 = 2j + 16 \rightarrow j^2 - 2j - 8 = 0 \rightarrow j = 4, -2$ , but since someone cannot have a negative age, the only possible value is 4.
26. **B.** There are  $12!$  permutations of Applications,  $2(11!)$  of them have the p's next to each other (just treat pp as one term that can be arranged two ways internally), so the probability is just  $\frac{1}{6}$ .
27. **B.** The sum of the down distances is  $\frac{40}{1-0.9} = 400$ , and the up distances will be the same with the exception of the original 40 feet down, so the total is 760.
28. **D.** Boat A exerts a northward force of  $x = 50 \cos(60^\circ) = \frac{50\sqrt{3}}{2}$ , so in order to have Boat B exert a northward force of  $-\frac{50\sqrt{3}}{2}$ , the total force needs to be  $-\frac{50\sqrt{3}}{2 \sin(-30^\circ)} = 50\sqrt{3}$ .
29. **B.** Circle is of the form  $(x-a)^2 + (y-b)^2 = r^2$ , so  $(50-a)^2 + (50-b)^2 = r^2$ ,  $(20-a)^2 + (60-b)^2 = r^2$ ,  $(70-a)^2 + (10-b)^2 = r^2$ , which is a system of three equations and three unknowns. Solving for  $r^2$  gives 2500, so  $r = 50$ .
30. **B.** In expectation, you get  $30(0.9) = 27$  questions right, which means 3 wrong, and 27 right and 3 wrong is a 105.