

1. **A.** Note that for any circle is uniquely defined by its radius and center, so any circle that can be written in that form can also be written as $(x-h)^2 + (y-k)^2 = r^2$, so each point we know that satisfies that equation has three unknowns: h, k and r. So we have a system of 3 unknowns and as many equations as we have points given, so in order for there to be a unique solution, there must be at least 3 points.
2. **B.** The discriminant of $b^2 - 4ac = -8 < 0$ so the conic is an ellipse.
3. **B.** It is easy to see that two hyperbolas can intersect as few as zero times, namely $xy = 1$ and $xy = 2$. There is obviously no pair (x,y) such that their product is both 1 and 2. As for the maximum, consider the $y^2 - x^2 = 1$ and $x^2 - \frac{1}{9}y^2 = 1$, so taking the sum gives $\frac{8}{9}y^2 = 2 \rightarrow y = \pm \frac{3}{2}$ and plugging either solution into either equation gives $x^2 = \frac{5}{4} \rightarrow x = \pm \frac{\sqrt{5}}{2}$, so there are 4 intersections, $(\frac{\sqrt{5}}{2}, \frac{3}{2})$ and $(-\frac{\sqrt{5}}{2}, \frac{3}{2})$.
The solution to #30 shows that 5 points uniquely defines a conic, which means that no distinct two conics can intersect 5 times or more, so 4 must be the max.
4. **C.** $y = ax^2 + bx + c$ for all points on the parabola, so $a + b + c = 5$,
 $4a + 2b + c = 10$, $9a + 3b + c = 17$ so $a = 1$, $b = 2$, $c = 2$ therefore, when $x = 4$,
 $y = 26$
5. **B.** The area of an ellipse is $ab\pi$, where a = the length of the semimajor axis and b = the length of the semiminor axis, so in this case, the area is $3 * 2 * \pi = 6\pi$
6. **A.** The eccentricity of an ellipse is the focal length divided by the length of the semimajor axis, where the focal length = $\sqrt{a^2 - b^2} = \sqrt{9 - 4} = \sqrt{5}$, so $e = \frac{\sqrt{5}}{3}$.
7. **A.** Since the two lines are parallel, their distance is the difference of their constants over $\sqrt{a^2 + b^2}$, so $\left| \frac{5-10}{\sqrt{5}} \right| = \sqrt{5}$
8. **C.** Due the curvature of a sphere, the angles of a triangle on it will sum to more than 180 degrees, consider the situation where you're on the equator and walk east along the equator a quarter of the distance, then turn 90 degrees left, walk to the north pole, then turn left 90 degrees again and walk back to your starting point. When you get there, you will be facing due south, so to get back to your original orientation, you need to turn 90 degrees again, so the measure of that angle is also 90 degrees, so the sum of the angles in this triangle is 270 degrees.
9. **C.** $\cos \theta = \frac{x}{r} \rightarrow r^2(4 + 5 \cos^2 \theta) = 9x^2 + 4y^2 = 36 \rightarrow \frac{x^2}{4} + \frac{y^2}{9} = 1 \rightarrow a = 3, b = 2$
and as we saw in problem 5, the area of this ellipse is 6π .
10. **A.** Since $a = 3$, $b = 2$ as we showed above, and $c = \sqrt{a^2 - b^2}$ for an ellipse,
 $c = \sqrt{5}$

11. **B.** The focus of a circle is its center. A circle is just a special case of an ellipse, and it is what happens when the foci are in the same place, so focal length is 0, so $a = b = r$.
12. **B.** By definition, the major axis is the line segment containing the foci and going to the ends of the ellipse, so it must contain the foci. The minor axis, however, is perpendicular to the major axis and only intersects in the center, so unless the ellipse is a circle, the foci do not fall on the minor axis. Finally, the directrix lines of an ellipse are outside of the ellipse so there is no way the foci, which are inside, are on them.
13. **C.** $1 < r < 2$ is an annulus where the outer radius is 2 and the inner radius is 1, so the area in between is $2^2\pi - 1^2\pi = 3\pi$ and the $0 < \theta < \pi$ constraint just halves the area, so the area is $\frac{3\pi}{2}$.
14. **C.** The focal length of this conic is 1, and the length of the latus rectum is 4 times the focal length, so it is 4.
15. **B.** The vertex of a parabola is always half way between the focus and the directrix on a line perpendicular to the directrix, in this case, that line is $y = 0$, so the directrix must be of the form $x = k$, this means the distance from the directrix to the vertex is $|k|$, while the distance from the focus to the vertex is 1, so $k = \pm 1$ but since the focus is at $(1, 0)$ the directrix cannot be $x = 1$, so it must be $x = -1$.
16. **C.** $0 \leq z \leq 1, x^2 + y^2 \leq z^2$ is a cone of height 1 with base area π so that has a volume of $\frac{\pi}{3}$, $-1 \leq z \leq 0, x^2 + y^2 \leq z^2$ is just $0 \leq z \leq 1, x^2 + y^2 \leq z^2$ flipped over the z-axis, so its volume is $\frac{\pi}{3}$ as well, so the total volume is $\frac{2\pi}{3}$.
17. **B.** If the directrix is $y = -1$, the line containing the focus and the vertex must be $x = k$ so it is perpendicular to the directrix, and since it contains the point $(0, 0)$, k must be 0, so the focus must be $(0, c)$ and since the distance from the directrix to the vertex is 1, the distance from the focus to the vertex must also be 1, so $c = 1$, so the focus is $(0, 1)$, this means all points on the parabola are equidistant from the line $y = -1$ and the point $(0, 1)$ and the only one of the points listed that is is $(4, 4)$.
18. **A.** The diameter of the larger sphere is the diagonal of the cube, which is $3\sqrt{3}$, and the diameter of the smaller sphere is the side of the cube, which is 3, and since $V = \frac{1}{6}\pi d^3$, the difference is $\frac{\pi}{6}27(3\sqrt{3} - 1) = \frac{9}{2}\pi(3\sqrt{3} - 1)$
19. **C.** Area of a triangle is with sides of length a, b, c and semiperimeter s is $\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{(15)(4)(5)(6)} = 30\sqrt{2}$
20. **D.** The eccentricity of the hyperbola is independent of the slopes of the asymptotes, so their sum is free to be any value (note that the sum isn't always zero, that is just the case when the hyperbola isn't rotated, by rotating you can make the sum of the slopes be whatever you want)
21. **C.** $x + 1 = t, y + 1 = \frac{1}{t} \rightarrow (x + 1)(y + 1) = 1$ which is a hyperbola.

22. **E.** In rectangular coordinates, those points are $(7, 0)$ and $(5, 0)$ so the midpoint is $(6, 0)$ which is not congruent to any of the choices.
23. **A.** The circle given has center $(3, -3)$ and radius 3. Since the distance between $(3, -3)$ and $(6, 1)$ is 5, and the circle extends 3 in every direction, the circle gets as close as 2 to the point.
24. **C.** A polar rose $r = A \cos(k\theta)$ has $2k$ petals if k is even, and k petals otherwise, so the first one has 12 petals, while the second has 3, so the difference is 9.
25. **D.** The graph is known as the Spiral of Archimedes.
26. **A.** The point $(5, 0)$ satisfies the equation $r = 5 \sec(\theta)$.
27. **B.** From high above on the z -axis you only see the x - y plane, so it looks like a circle, and from high above on the y -axis you only see the x - z plane, so it looks like an ellipse.
28. **B.** Say the vertices of the figure have coordinates (x_1, y_1, z_1) , (x_2, y_2, z_2) , and

(x_3, y_3, z_3) , then its centroid is at $(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3})$.

Note that the midpoints of the sides of the figure are $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2})$,

$(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}, \frac{z_1 + z_3}{2})$, and $(\frac{x_3 + x_2}{2}, \frac{y_3 + y_2}{2}, \frac{z_3 + z_2}{2})$ and that the average

of the coordinates of these points is $(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3})$,

which is the centroid. So we can just average the given midpoints to find the centroid, which is $(-1, 2, 3)$, so the sum is 4.

29. **D.** The area of a triangle with vertices $(0, 0)$, $(1, 1)$ and (x, y) is:

$$\begin{array}{cccc} & 0 & 0 & \\ 0 & 1 & 1 & 0 \\ x & x & y & y \\ \hline 0 & 0 & 0 & 0 \\ x & & & y \end{array}$$

$$\left| \frac{x-y}{2} \right| = 17 \rightarrow \frac{x-y}{2} = 17 \text{ or } \frac{x-y}{2} = -17, \text{ and the graph of that is two lines.}$$

30. **C.** All 2d conics can be written in the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, but this means they can also be written as $\frac{Ax^2 + Bxy + Cy^2 + Dx + Ey + F}{A} = 0$, or
- $$x^2 + \frac{B}{A}xy + \frac{C}{A}y^2 + \frac{D}{A}x + \frac{E}{A}y + \frac{F}{A} = 0 \text{ or } x^2 + B'xy + C'y^2 + D'x + E'y + F' = 0$$
- and since each point is another equation of these 5 unknowns, you need at least 5 of them to have a unique solution. Note that if $A = 0$, then just divide by some other nonzero constant instead.