1) Via conversion, the mean time is 11916 seconds and the standard deviation is 1394 seconds. Now the bottom $\frac{30}{3750} = .8\%$ will advance to the state marathon. Now, $P(z \le Z) = 0.008$ corresponds to a Z-value of -2.41. The Z-value for this specific problem is $Z = \frac{x - 11916}{1394}$, where x is the maximum time that will advance to the state final. So $\frac{x - 11916}{1394} = -2.41 \Rightarrow x = 8556.46$ seconds, which rounds to 8556 seconds or 142 minutes and 36 seconds.

2) Notice first that there are 99 people in the table.

A: The number of people who drive a brown car are 7+3+4=14. So $P(\text{browncar}) = \frac{14}{99}$

B: $P(\text{Brown} \mid \text{Blonde}) = \frac{P(\text{brown} \cap \text{blonde})}{P(\text{blonde})} = \frac{\frac{4}{99}}{\frac{21}{99}} = \frac{4}{21}$

C: Notice there are only 64 members who drive blue or black cars. Of this 64, 4 have red hair. Thus P(red hair|(blue or black car)) = $\frac{4}{64} = \frac{1}{16}$

D: Notice that 20 people drive a car that is the same color as their hair. Of these 20, 9 have black hair. Thus P(black hair|car same color as hair) = $\frac{9}{20}$.

So $\frac{AD}{BC} = \frac{\frac{14}{99}(\frac{9}{20})}{\frac{4}{21}(\frac{1}{16})} = \frac{294}{55}$

3) The total probability is a triangle with height of 4 and a base of 40, for an area of 80. $P(0 < X < 6)$ is a trapezoid of height 6 and bases of length f(0) and f(6).

$$P(0 < X < 6) = \frac{\frac{6}{2}\left(f(0) + f(6)\right)}{80}$$

$$P(0 < X < 6) = \frac{3\left(4 + \frac{17}{5}\right)}{80}$$

$$P(0 < X < 6) = \frac{\frac{111}{5}}{80} = \frac{111}{400}$$

4) Mean CAN be negative (for example, if all the numbers in the set are negative). Median CAN be negative (same reason as the mean). Standard Deviation CANNOT be negative (it is the positive square root of the variance). Correlation CAN be negative (this is the square root of the coefficient of determination). Coefficient of Determination CANNOT be negative (since the correlation is the square root of this number it must be positive). Variance CANNOT be negative (this is the square of standard deviation and thus must be positive). Mode CAN be negative (same reason as for the mean).

   Thus 4 values can be negative. We now want the probability that we get exactly 4 tails in a row if we flip a coin 7 times. First case is getting 4 tails in the first four flips. This can occur in 4 ways since we can get HHT,HTH, HTT, HHH after this (we cannot get T_ _ because then we would have more than 4 tails in a row). Repeat this process for exactly 4 tails in slots 2-5, 3-6, and 4-7. Repeat this process for getting exactly 5 tails, 6 tails, or 7 tails in a row. The final answer is $\left(\frac{1}{2}\right)^7(4+2+2+4+2+1+2+2+1)=\frac{5}{32}$

5) Using the calculator, we get that A=12, B=84, C= -19, D= 168, E= -137. So
   $\frac{ACDE}{B}=\frac{12(-19)(168)(-137)}{84}=62472$

6) Use the Poisson Distribution. We know that the expected number of chips in each cookie is $\frac{2000}{200}=10$. So P(at least 13 chips) = 1-P(less than 13 chips) =

$$1-[\frac{10^0e^{-10}}{0!}+\frac{10^1e^{-10}}{1!}+\frac{10^2e^{-10}}{2!}+\frac{10^3e^{-10}}{3!}+\frac{10^4e^{-10}}{4!}+\frac{10^5e^{-10}}{5!}+\frac{10^6e^{-10}}{6!}+\frac{10^7e^{-10}}{7!}$$

$$+\frac{10^8e^{-10}}{8!}+\frac{10^9e^{-10}}{9!}+\frac{10^{10}e^{-10}}{10!}+\frac{10^{11}e^{-10}}{11!}+\frac{10^{12}e^{-10}}{12!}]=1-0.7916=0.2084$$

7) The sum of the residuals of a set of data points from the line of best fit is always 0.

8) I) If V is the variance of a data set X, $V(aX+b)=a^2V(X)$ II) Notice that the "b" does not change the variance (and therefore, not the standard deviation). III) Taking the log will change the standard deviation. IV) See part I. V) The "a" is -1, but $(-1)^2=1$, S.D. does not change. VI) $\sigma=\sqrt{\frac{\sum(x-\bar{x})^2}{n}}$. Although the numerator will not change, n does, so S.D. does.

   I, III, IV, VI.

9) Think about this in terms of sums, an odd sum can be garnered by getting (odd+odd+odd) or (odd+even+even) in any order. So, keeping that in mind, look at the individual probabilities. P(small black) = $\frac{1}{100}$, P(gray) = $\frac{9-1}{100} = \frac{8}{100}$, P(small white) = $\frac{25-9}{100} = \frac{16}{100}$, P(big black) = $\frac{36-25}{100} = \frac{11}{100}$ and P(big white) = $\frac{100-36}{100} = \frac{64}{100}$. Now from this we get an even score when we hit a white or black area, so P(even) = $\frac{92}{100} = \frac{23}{25}$ and we get an odd score when we hit a gray area, so P(odd) = $\frac{8}{100} = \frac{2}{25}$. So P(odd sum) = P(3 odds) + P(2 evens, 1 odd) = $(\frac{2}{25})^3 + ({}_3C_2)(\frac{23}{25})^2(\frac{2}{25}) = 0.20$

10) If 8 is to be the second largest number, then 4 numbers must be less than 8 and 1 must be larger. Out of the first 10 natural numbers, 7 are less than 8 and we want to choose 4. This can be done in ${}_7C_4 = 35$ ways. Also, 2 numbers are greater than 8 and we want to choose 1. This can be done in ${}_2C_1 = 2$ ways. So the number of combinations of 6 numbers where 8 is the second largest is 35(2)=70. So the probability is $\frac{70}{{}_{10}C_6} = \frac{70}{210} = \frac{1}{3}$

11) P(1 day healing) = $\frac{6-1}{15} = \frac{1}{3}$, P(2 days healing) = $\frac{6-2}{15} = \frac{4}{15}$, P(3 days healing) = $\frac{6-3}{15} = \frac{1}{5}$, P(4 days healing) = $\frac{6-4}{15} = \frac{2}{15}$, P(5 days healing) = $\frac{6-5}{15} = \frac{1}{15}$

So E(payment) = $\frac{1}{3}(5) + \frac{4}{15}(10) + \frac{1}{5}(15) + \frac{2}{15}(17.5) + \frac{1}{15}(20) = 11$

12) The outcomes for rolling 2 dice are {(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}. So the lowest number is one 11 times, two 9 times, three 7 times, four 5 times, five 3 times, six 1 time. So the expected value is E =

$$\frac{11}{36}(1) + \frac{9}{36}(2) + \frac{7}{36}(3) + \frac{5}{36}(4) + \frac{3}{36}(5) + \frac{1}{36}(6) = \frac{91}{36}$$

13) A: ${}_{10}C_1 + \ldots + {}_{10}C_{10} = 2^{10} - 1 = 1023$

B: Notice that ${}_{n}C_1 = \dfrac{n!}{(n-1)!1!} = n$. So the sum is just 1+2+…+10=55

C: First reduce: $\dfrac{{}_{14}C_r}{{}_{14}C_{(r-1)}} = \dfrac{\dfrac{14!}{(14-r)!r!}}{\dfrac{14!}{(14-r+1)!(r-1)!}} = \dfrac{(15-r)!(r-1)!}{(14-r)!r!} = \dfrac{15-r}{r}$

So the product becomes $\prod_{r-2}^{14} \dfrac{15-r}{r} = (\dfrac{13}{2})(\dfrac{12}{3})\ldots(\dfrac{3}{12})(\dfrac{2}{13})(\dfrac{1}{14}) = \dfrac{1}{14}$

D: First reduce: $\dfrac{{}_{14}P_r}{{}_{14}C_r} = \dfrac{\dfrac{14!}{(14-r)!}}{\dfrac{14!}{(14-r)!r!}} = r!$. So the product becomes

$\prod_{r=1}^{5} r! = 1!(2!)(3!)(4!)(5!) = 34560$

So $\dfrac{AD+BC}{BD} = \dfrac{1023(34560)+55(\dfrac{1}{14})}{55(34560)} = 18.6$

14) The probability of getting no pregnant hamster is $\frac{49}{50}$. Since there is no replacement, the probability of getting no pregnant hamster is $\frac{48}{49}$. Following this trend, we get that

$\frac{49}{50} \cdot \frac{48}{49} \cdot \frac{47}{48} \cdot \frac{46}{47} \cdot \frac{45}{46} = \frac{9}{10}$

15) Median is 3 – it's the middle of the data.
Mode is 4 – the data that shows up the most.
Mean is 3 – the sum of the data over the sum of the population.

$\sqrt[3]{4^3} = 4.$