

1. The lowest score is from getting them all wrong (-30) and the highest is from getting them all right (120). The sum of these is 90. **B**

2. For any question that you answer, there's a  $1/5$  chance you earn 4 points and a  $4/5$  chance you earn -1 points. The expected score for any answered question would then be  $(1/5)(4) + (4/5)(-1) = 0$ . Since unanswered questions are also worth 0, the expected total score will be 0. **A**

3. It is easier to think of your score for this as 120 with -4 for each unanswered question and -5 for each wrongly-answered question. The lowest unobtainable score would then be the 120 minus the highest unobtainable combination of 4 and 5. The highest number that cannot be represented as a combination of 4's and 5's is  $(4)(5) - 4 - 5 = 11$ . Hence the lowest unobtainable score is  $120 - 11 = 109$ . **C**

4. A score of 1 from 4 questions is 1 correct and 3 incorrect. The only lower score would be from 4 incorrect. The probability that you get all four incorrect is  $(4/5)^4$ . The probability that you get 1 correct and 3 incorrect is  $(1/5)(4/5)^3$ . Since there are 4 different questions you could get correct, the total probability is  $(4/5)^4 + 4(1/5)(4/5)^3 = 512/625$ . **D**

5. A score of 100 is obtained by either answering 25 correctly and leaving 5 unanswered, or 26 correct and 4 incorrect answers. The number of combinations of choosing 5 questions not to answer is  $30C5$ . Similarly, the number of combinations of choosing 4 questions to answer incorrectly is  $30C4$ . However, for *each* of the 4 incorrect questions, you can choose any of 4 answers, so the total number of combinations is  $30C5 + 30C4 * 4 * 4 * 4 * 4 = 7,158,186$ . **D**

6. In order to find the oblique asymptote, divide  $x + 2$  into  $x^3 - 5x + 3$  and ignore the remainder. Performing the long division yields  $x^2 - 2x - 1$ . **C**

7. Square both sides of the equation to get  $(1+x) + (2-x) + 2\sqrt{(1+x)(2-x)} = 4 = 3 + 2\sqrt{(1+x)(2-x)}$ . Hence  $\sqrt{(1+x)(2-x)} = (4-3)/2 = 1/2$ . **B**

8.  $A = 2$ .  $L = \sqrt{3^2 + 4^2} = 5$ .  $P = (t^4)^{502} = t^{2008} = 1$ .  $H = 1$ . Hence ALPHA =  $(2)(5)(1)(1)(2) = 20$ . **D**

9. Combining the logs gives you  $f(x) = \log_2(\sin(x)\cos(x)\cos(2x)\cos(4x))$ .  $\sin(x)\cos(x) = \sin(2x)/2$ . Therefore,  $f(x) = \log_2(1/2 * \sin(2x)\cos(2x)\cos(4x))$ . Similarly,  $\sin(2x)\cos(2x) = \sin(4x)/2$ , and therefore  $f(x) = \log_2(1/4 * \sin(4x)\cos(4x))$  and similarly,  $f(x) = \log_2(1/8 \sin(8x))$ .  $f(\pi/16) = \log_2(1/8 \sin(\pi/2)) = \log_2(1/8) = -3$ . **B**

10.  $2008331^2 - 2008329^2 = (2008331 - 2008329)(2008331 + 2008329) = 2(4016660) = 8033320$ . **C**

11.  $f(1) = (\sqrt{0 + 1})^2 = 1$ .  $f(2) = (\sqrt{1 + 1})^2 = 4$ .  $f(3) = (\sqrt{4 + 1})^2 = 9$ . And you can see that  $f(n) = n^2$ . Hence,  $f(10) = 100$ . **C**

12. Since  $f(0)*f(1)*f(2) = f(1)*f(2)*f(3)$ , you can see  $f(0)=f(3)$ . Similarly, you can see  $f(3)=f(6)$  and, in general  $f(3n) = f(0)$ . Also similarly, you can see  $f(3n+1)=f(1)$ , and  $f(3n+2)=f(2)$ .  $f(7)=f(1)=5$ , and  $f(15)=f(0)=2$ . Since  $f(0)*f(1)*f(2) = 30$ ,  $f(2) = 3$ . Since  $f(38) = f(2)$ ,  $f(38) = 3$ . **B**

13. To start off, note  $f(x)$  and  $g(x)$ , (and hence  $f^{[n]}(x)$  and  $g^{[n]}(x)$ ) are inverses of each other. We're given that  $g^{[10]}(x) = h^{[15]}(0)$ . Then take  $h^{[10]}(g^{[10]}(x)) = h^{[10]}(h^{[15]}(0)) \rightarrow x = h^{[25]}(0)$ . **A**

14.  $100(1) + 99(2) + \dots + 52(49) + 51(50) + 51(51) + 52(52) + \dots + 99(99) + 100(100) =$   
 $100(101) + 99(101) + 98(101) + \dots + 52(101) + 51(101) = 101(100 + 99 + 98 + \dots + 53 + 52 + 51) =$   
 $101(100 + 51)(50/2) = 101(151)(25)$ . Evaluate this value mod 6.  $(101)(151)(25) \equiv (5)(1)(1) \equiv 5 \pmod{6}$  **D**
15. The first nine palindromic numbers are 1-9. The next nine are 11, 22, ..., 99. The 19th is 101, and the 20th is 111. **C**
16.  $\sqrt{(9-x^2)} = \sqrt{[9 - (3\sin\theta)^2]} = \sqrt{[9 - 9\sin^2\theta]} = \sqrt{[9(1-\sin^2\theta)]} = \sqrt{9\cos^2\theta} = 3|\cos\theta|$ . **D**
17.  $\pi$  rad = 180 deg so 1 deg =  $\pi/180$  rad. 300 deg =  $300*\pi/180$  rad =  $5\pi/3$  rad. **C**
18. Putting the right side under a common denominator, you can see  $6-x = A(x+2) + B(x-2)$ . Since this is true for all  $x$ , it's true for any specific  $x$ , like  $x=2$ . If  $x=2$ , then  $4 = 4A$  and  $A=1$ . If  $x=-2$ , then  $8 = -4B$  and  $B=-2$ .  $AB = (1)(-2) = -2$ . **A**
19. The slope between  $(3,14)$  and  $(3+h, f(3+h))$  is  $[f(3+h) - 14]/h = [(3+h)^2 + (3+h) + 2 - 14]/h$ , which, after expanding, is  $[(9 + 6h + h^2) + (3 + h) + 2 - 14]/h = (7h + h^2)/h = 7 + h$ . As  $h \rightarrow 0$ , the slope is 7. **C**
20. =====. The limit as  $n \rightarrow \infty$  is  $2/6 + 1/2 + 2/1 = 17/6$ . **B**
21.  $\text{GCF}(p, p + k) = 1$ , unless  $p$  is equal to one of the prime factors of  $k$ , in which case  $\text{GCF}(p, p + k) =$  that particular prime factor of  $k$ . The largest prime factor of 66 is 11, so letting  $p = 11$  gives  $\text{GCF}(11, 77) = 11$ . **E**
22.  $2(2) + 3(-1) + 1(1) = 2$ . **B**
23. If the number does not look familiar to you, read it as  $(1 + 1/1000)^{1000}$ , which would come up in taking the limit of  $(1 + 1/x)^x$  as  $x \rightarrow \infty$ , which is the number  $e$ .  $(1001/1000)^{1000}$  is actually very close to  $e$ , which is about 2.7. This number is closest to 3. **C**
24. You could solve for the matrix  $\mathbf{X}$ ; however, it is simply easier to take the determinate of both sides, giving you  $\det(\mathbf{X}) * [(-1)(-4) - (2)(3)] = [(2)(0) - (8)(1)]$ . This is  $\det(\mathbf{X}) * (-2) = -8$ , and  $\det(\mathbf{X}) = 4$ . **D**
25.  $y = 2\sin(\pi x - 3)\cos(\pi x - 3) = \sin(2\pi x - 6)$ . This has amplitude 1. **A**
26. Writing -1 in polar form gives  $e^{\pi i}$ .  $(-1)^{-i}$  is then  $(e^{\pi i})^{-i} = e^{\pi}$ . Then  $(e^{\pi})^{1/\pi} = e$ . **E**
27. What's inside the parentheses is clearly not 0 and seems reasonably finite (and, in fact, is equal to exactly  $\pi^2/6$ ). So raising it to the 0th power gives 1. **A**
28. If  $\text{Arccoth}(2) = x$ , then  $\text{coth}(x) = 2$ .  $\text{coth}(x) = (e^x + e^{-x})/(e^x - e^{-x}) = 2$ . And  $e^x + e^{-x} = 2e^x - 2e^{-x}$ . Bringing everything to one side gives  $e^x - 3e^{-x} = 0$ . Factoring  $e^{-x}$  out of the left side gives  $e^{-x}(e^{2x} - 3) = 0$ . Then, either  $e^{-x} = 0$  or  $e^{2x} - 3 = 0$ . The first clearly cannot happen, and to solve the second one, say  $e^{2x} = 3$ ,  $2x = \ln(3)$  and  $x = 1/2 * \ln(3)$ . **C**
29. Let  $x =$ . Since it is infinite, this is equivalent to saying  $x =$ . Solving this by squaring both sides and bringing everything to the left side gives  $x^2 + x - 2 = 0$ . The solutions to this are  $x=1$  and  $x=-2$ . Since a squareroot cannot be negative,  $x=1$ . **B**

30. Factoring out the  $\sec^2(15)$  gives  $\sec^2(15)[1 + \cos(30)]$ . Noting that 30 is twice 15, you can apply the double-angle formula to get  $\sec^2(15)[1 + 2\cos^2(15) - 1] = \sec^2(15) * 2\cos^2(15) = 2$ . (The two angles are, in fact, 15 and 30 radians, not degrees, but ultimately, this does not matter in the answer.) **C**