T1. The angle equivalent to $15\pi/13$ with respect to sin on $[-\pi/2,\pi/2]$ is $-2\pi/13$, so $A = -2\pi$. Similarly, the angle equivalent to $15\pi/13$ with respect to cos on $[0,\pi]$ is $11\pi/13$, so $B = 11\pi$. For C and D, consider $\theta = \sin^{-1}(12/13)$. If you let θ be one angle of a right triangle, its opposite side must be 12 and its hypotenuse must be 13, and hence its adjacent side must be $\sqrt{(13^2 - 12^2)} = 5$. Then the $\sin(\theta) = 12/13$ and the $\cos(\theta) = 5/13$, and hence $C = 12\pi$ and $D = 5\pi$. A+B+C+D = **26π**.

T2. This is equivalent to asking what the product of g(x)'s roots are if g(x)'s roots are all 1 more than f(x)'s roots. We can construct g(x) as f(x-1). Furthermore, for a quintic polynomial, the product of its roots is the negative of the constant term divided by the leading coefficient, so we only need to find these two terms. Since $g(x) = (x-1)^5 - 2(x-1)^4 + 4(x-1)^3 + 7(x-1)^2 - 6(x-1) + 3$, if we only consider the x^5 and constant terms of each expansion we get $x^5 + (-1)^5 - 2(-1)^4 + 4(-1)^3 + 7(-1)^2 - 6(-1) + 3 = x^5 + 9$, and hence the leading term is $1x^5$ and the constant term is 9. $-9/1 =$ **-9**.

T3. $A = \lim_{x\to 1} (x-1) = (1-1) = 0$ since (x-1) is continuous.

$$B = \lim_{x\to 1}\left(\frac{x^2+x-2}{x-1}\right) = \lim_{x\to 1}\left(\frac{(x+2)(x-1)}{x-1}\right) = \lim_{x\to 1}(x+2) = (1+2) = 3$$

$$C = \lim_{x\to\infty}\left(\frac{2x^2-x+12}{x^2+3x-5}\right) = \lim_{x\to\infty}\left(\frac{2-1/x+12/x^2}{1+3/x-5/x^2}\right) = \frac{2-0+0}{1+0-0} = 2$$

$$D = \lim_{x\to\infty}\left(\frac{4x^2-10x+7}{2x^3-5x+1}\right) = \lim_{x\to\infty}\left(\frac{4/x-10/x^2+7/x^3}{2-5/x^2+1/x^3}\right) = \frac{0-0+0}{2-0+0} = 0$$

A+B+C+D = **5**.

T4. The restrictions on the domain of f(x) are that the bottom of a fraction cannot be 0 and that the inside of a square root cannot be negative. Hence the domain of f(x) is $-x^2 + 4x + 12 > 0$. Factoring, we get $-(x+2)(x-6) > 0$. Since the leading term is negative, it opens downward, and hence it is positive on (-2, 6), and A = -2, B = 6.
Since g(x) is a parabola that opens up, its minimum will occur at its vertex (assuming the vertex is in the domain) and its maximum will occur at the endpoint of the domain that is furthest from the vertex. Recalling that the vertex of a parabola occurs at $x = -b/2a$, the vertex for g(x) occurs at x = -3, and so the minimum is g(-3) = 6. The maximum will occur at the domain endpoint furthest from -3, which is x = 1, and the maximum is g(1) = 22. So C = 6 and D = 22. A+B+C+D = **32**.

T5. A rough sketch of these functions will provide the answer easily. f(x) and g(x) intersect twice (once near x=0 and once near x=1.5). f(x) and h(x) intersect twice (once at x=-1 and once at x=2). f(x) and i(x) intersect twice (once at x=-2 and once at x=2), but we have already counted the intersection at x=2, so we don't count it again. g(x) and h(x) clearly don't intersect (as $\ln(x) < x$ for all x). g(x) and i(x) intersect once (at $x=e^2$). h(x) and i(x) once (at x=2), but we have already counted this, so we don't count it again. Hence there are a total of 2+2+1+1 = **6** unique intersections.

T6. In parallel, there's a $(4C0)*(1/2)^4$ chance that none of them will work, a $(4C1)*(1/2)^3*(1/2)$ chance that exactly one will work, a $(4C2)*(1/2)^2*(1/2)^2$ chance that exactly two will work, a $(4C1)*(1/2)*(1/2)^3$ chance that exactly three will work, and a $(4C4)*(1/2)^4$ chance that all four will work. Hence the expected number of working lights is (1/16)*0 + (4/16)*1 + (6/16)*2 + (4/16)*3 + (1/16)*4 = 2.

In series, the chance that none will work is the chance that the first one doesn't (1/2), the chance that exactly one will work is the chance that the first one does and the second one doesn't (1/2)(1/2), the chance that exactly two will work is the chance that the first two work and the third doesn't $(1/2)^2(1/2)$, the chance that exactly three will work is the chance that the first three work and the fourth doesn't $(1/2)^3(1/2)$, and the chance that all work is simply $(1/2)^4$. Hence, the expected number of working lights is (1/2)*0 + (1/4)*1 + (1/8)*2 + (1/16)*3 + (1/16)*4 = 15/16. The difference is **17/16**.

T7. The parallelogram can be defined with the two vectors **u** = <3,1> and **v** = <1,2> from the point (1,2). The area of a parallelogram defined by two vectors is |**u** x **v**|. **u** x **v** = <0,0,3*2-1*1> = <0,0,5>, whose magnitude is 5 = A.
The diameter has length $\sqrt{(7-1)^2 + (11-3)^2} = \sqrt{6^2 + 8^2} = 10$. So the radius is 5, and the area is 25π = B.
Sketching these two graphs shows that it is a square whose diagonals are of length 2, and hence whose sides are $2/\sqrt{2} = \sqrt{2}$. The area is then $(\sqrt{2})^2 = 2$ = C.
For a 30-60-90 triangle, the side opposite the 30 angle is half the hypotenuse (i.e. 1) and the other leg is the short leg times $\sqrt{3}$ (i.e. $\sqrt{3}$). The area is then $(1/2)(1)(\sqrt{3}) = \sqrt{3}/2$ = D.
ABCD = **$125\pi\sqrt{3}$**.

T8. If $A(x) = ax^2 + bx + c$, and A(0) = 2, then $c = 2$. If the product of its roots is 1, then $c/a = 1$, and hence $a = 2$. If A(1) = -1, then $a + b + c = -1$, and hence $b = -5$.
If B(x) is a cubic with roots -1, 0 and 2, then $B(x) = ax(x+1)(x-2)$ for some a. If B(1) = -6 then $-2a = -6$, and $a = 3$.
If C(x) is a polynomial of least degree with all real coefficients, and $1 + i$ is a zero, then $1 - i$ must be the only other zero. So $C(x) = a(x - (1 + i))(x - (1 - i)) = a(x^2 - 2x + 2)$. If C(0) = 2, then $2a = 2$ and $a = 1$.
C(2) = 2, A(C(2)) = A(2) = 0, B(A(C(2))) = B(0) = **0**.

T9. $$A = \log_9\left(\frac{6}{5}\right) + \log_9\left(\frac{10}{3}\right) = \log_9\left(\frac{6}{5}\cdot\frac{10}{3}\right) = \log_9(4) = \frac{\log(4)}{\log(9)} = \frac{\log(2^2)}{\log(3^2)} = \frac{\log(2)}{\log(3)}.$$

If $3^B = 16$, then $B = \log_3(16) = \frac{\log(16)}{\log(3)} = \frac{\log(2^4)}{\log(3)} = \frac{4\log(2)}{\log(3)}$.

$$C = \log_{125}(8)\cdot\log_{256}(25) = \frac{\log(8)}{\log(125)}\cdot\frac{\log(25)}{\log(256)} = \frac{\log(2^3)}{\log(5^3)}\cdot\frac{\log(5^2)}{\log(2^8)} = \frac{3\log(2)}{3\log(5)}\cdot\frac{2\log(5)}{8\log(2)} = \frac{1}{4}.$$

$$\frac{A}{B} = \frac{\log(2)}{\log(3)}\cdot\frac{\log(3)}{4\log(2)} = \frac{1}{4}. \qquad \frac{A}{B} + C = \frac{1}{4} + \frac{1}{4} = \mathbf{\frac{1}{2}}.$$

T10. For A and B, complete the square: $(x^2 - 2x + 1) + 9(y^2 + 4y + 4) = -1 + 1 + 9(4)$ to get $(x - 1)^2 + 9(y + 2)^2 = 36$ and $\frac{(x-1)^2}{36} + \frac{(y+2)^2}{4} = 1$. So the conic is an ellipse with a=6 and b=2.
The focal radius is $\sqrt{a^2 - b^2} = \sqrt{32} = 4\sqrt{2}$ and so the distance between the foci is $2*4\sqrt{2} = 8\sqrt{2}$ = A.
The length of the latus rectum is $2b^2/a = 8/6 = 4/3$ = B.
For C and D, complete the square $4(x^2 + 2x + 1) - (y^2 - 6y + 9) = 41 + 4(1) - 1(9)$ to get $4(x + 1)^2 - (y - 3)^2 = 36$ and $\frac{(x+1)^2}{9} + \frac{(y-3)^2}{36} = 1$. The intersection of the asymptotes is the center of this hyperbola, (-1, 3), so C = -1, D = 3, and ABCD = **$-32\sqrt{2}$**.

T11. If $a_1 + a_2 + ... + a_9 + a_{10} = \sin(10\pi/6) = -\sqrt{3}/2$ and $a_1 + a_2 + ... + a_9 = \sin(9\pi/6) = -1$, then $\sin(10\pi/6) - \sin(9\pi/6) = a_{10} = -\sqrt{3}/2 + 1$.
b_n is just the sum of the first n squares, so $b_n = n(n+1)(2n+1)/6$, and so $b_{10} = 10(11)(21)/6 = 385$.
To evaluate the limit of c_n as $n \to \infty$, you can either multiply it by its conjugate over its conjugate, or simply notice that $\sqrt{n^2+4n} - \sqrt{n^2+2n} = \sqrt{n^2+4n+4} - \sqrt{n^2+2n+1}$ since as n becomes arbitrarily large, a constant inside of the square root becomes insignificant. That means $c_\infty = (n+2) - (n+1) = 1$.
A+B+C = **387 - √(3)/2**.

T12. The first triangle has legs of length √2, and hence an area of √(2)*√(2)/2 = 1. The legs of the second triangle will have length 1, and hence an area of 1*1/2 = 1/2. The legs of the third triangle will have length 1/√2, and hence an area of 1/√2 * 1/√2 * 1/2 = 1/4. You can see the areas form a geometric sequence whose first term is 1 and whose ratio is 1/2. Hence the sum of all the areas is 1/(1-1/2) = 2. To find the overall perimeter, you do *not* want to add up the individual perimeters, since two of each triangle's sides (the hypotenuse and one leg) are part of the interior of the shape (except for the first one). Hence, you want to add up one leg from each triangle and the hypotenuse of the first one. The first triangle's legs are √2, the second 1, the third 1/√2, etc. So they form a geometric series with first term √2 and ratio 1/√2 and the sum is √2/(1-1/√2) = 2/(√2 – 1) = [2(√2 + 1)] / [(√2 -1)(√2 + 1)] = 2√2 + 2. Add in the first hypotenuse (the only "unused" one) to get a total perimeter of 2√2 + 4.
The sum of the area and perimeter is then **2√2 + 6**.

T13. This will be true in three cases: when the exponent is 0 and the base is nonzero, when the base is 1, and when the base is -1 and the exponent is even.
The exponent is 0 at x = -2 and x = -3, but at -2, the base is also 0, so the only solution from this case is x = -3.
When the base is 1, $x^2 + 2x = 1$ and $x^2 + 2x - 1 = 0$, which has two distinct real (and irrational) solutions, the sum of which is -2/1 = -2.
When the base is -1, $x^2 + 2x = -1$ and $x^2 + 2x + 1 = 0$, which has the single solution of x = -1. At this point, the exponent is $(-1)^2 + 5(-1) + 6 = 2$, which is even, so x = -1 is a legitimate solution.
The sum of all of these solutions is -3 + -2 + -1 = **-6**.

T14. Use the $A = \frac{1}{\frac{1}{1}+\frac{1}{\frac{1}{2}}} = \frac{1}{3}$, $B = \frac{1}{\frac{1}{\frac{1}{2}}+\frac{1}{-2}} = \frac{2}{3}$, $C = \frac{1}{\frac{1}{1}+\frac{1}{\frac{1}{2}}+\frac{1}{-2}} = \frac{2}{5}$, so $\frac{1}{3}+\frac{2}{3}+\frac{2}{5} = \frac{7}{5}$.

T15. If $Q^{-1}(x) = x^2 - 2x$, then $x = Q^2 - 2Q = Q^2 - 2Q + 1 - 1 = (Q-1)^2 - 1$. Solving for Q gives $Q(x) = \sqrt{x+1} + 1$
Similarly, if $T^{-1}(x) = 2\sin^{-1}(-x/2) + 2\pi/3$, then $x = 2\sin^{-1}(-T/2) + 2\pi/3$, and $x/2 + \pi/3 = \sin^{-1}(-T/2)$, and $T(x) = -2\sin(x/2 + \pi/3)$.
$T(\pi) = -1$, and $Q(T(\pi)) = Q(-1) =$ **1**.