1. A: If the exterior angle is 24 then, the interior angle is 156. So

$$\frac{180(n-2)}{n} = 156 \Rightarrow 180n - 360 = 156n \Rightarrow 24n = 360 \Rightarrow n = 15$$

B: The largest integral interior angle for any n-gon is $179°$. So

$$\frac{180(n-2)}{n} = 179 \Rightarrow 180n - 360 = 179n \Rightarrow n = 360.$$

C: A decagon has 10 sides, so the number of diagonals is found by $\frac{10(10-3)}{2} = 35$.

So $A - \frac{B \bullet C}{180} = 15 - \frac{360(35)}{180} = -55$

2. A: Let r be the original radius and r* be the new radius. Then

$3(\frac{4}{3})\pi r^3 = \frac{4}{3}\pi(r^*)^3 \Rightarrow 3r^3 = (r^*)^3 \Rightarrow 3 = (\frac{r^*}{r})^3 \Rightarrow 3^{1/3} = \frac{r^*}{r}$. Thus r is changed by a factor of $3^{1/3}$.

B: Let h be the original height and h* be the new height. Then

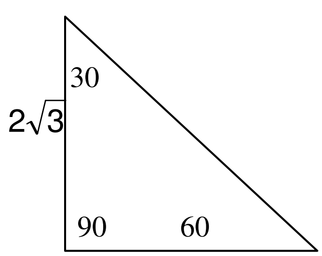
$4lwh = (2l)(\frac{2}{3}w)(h^*) \Rightarrow 4h = \frac{4}{3}h^* \Rightarrow 3 = \frac{h^*}{h}$. So the height is changed by a factor of 3.

C: Let s be the length of the side of the base and s* be the new side length. Then

$2(\frac{1}{3})s^2h = \frac{1}{3}(s^*)^2(4h) \Rightarrow 2s^2 = 4(s^*)^2 \Rightarrow \frac{1}{2} = \frac{(s^*)^2}{s^2} \Rightarrow \frac{\sqrt{2}}{2} = \frac{s^*}{s}$. So the length of the side of the base is changed by a factor of $\frac{\sqrt{2}}{2}$.

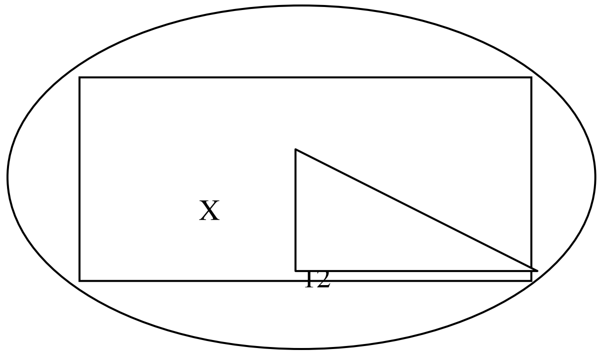
Then $A^6B^2C^2 = (\sqrt[3]{3})^6(3)^2(\frac{\sqrt{2}}{2})^2 = 9(9)(\frac{1}{2}) = \frac{81}{2}$

3. A: The triangle formed by the apothem and the half of the side of the triangle is



Using special triangle relations we can see that half of the side length of the hexagon is 2. So the side length of the hexagon is 4. So $A_{hex} = \frac{1}{2}(2\sqrt{3})(4 \bullet 6) = 24\sqrt{3}$

B:



The above diagram shows the rectangle inscribed in a circle with radius 13. Using the Pythagorean Theorem, we can get that x = 5. From this, we see that the width of the triangle is 10, so the area of the rectangle is 24(10)=240.

C: Using the special triangle relations again, we see that the height of the parallelogram is 3. So the area of the parallelogram is 3(10) = 30.

D: Since the circle has radius 12, the 24 in the perimeter of the sector comes from 2 of the radii. The 5π is the distance along the part of the circle. Notice that the circumference of the entire circle is $2\pi(12) = 24\pi$, so this sector is $\frac{5}{24}$ of the circle. Then, the area of the sector is $\frac{5}{24}[\pi(12)^2] = 30\pi$

Then $\frac{AC}{B} + \frac{D}{\pi} = \frac{24\sqrt{3}(30)}{240} + \frac{30\pi}{\pi} = 3\sqrt{3} + 30$

4. A: Using similar triangles by extending HI and FG so that they intersect and the fact that the parallels are equally spaced apart. Let the extension of FG to the intersection with HI be called y. Also call the length of one of the equal segments on the top of the figure be called x. Then $\frac{6}{y} = \frac{21}{5x+y} \Rightarrow 30x + 6y = 21y \Rightarrow y = 2x$. Using this info set up a second proportion to solve for the length of JK, which we will call a: $\frac{6}{2x} = \frac{a}{4x} \Rightarrow 24x = 2xa \Rightarrow a = 12.$

B. Notice that each of the quadrilaterals in the figure is a trapezoid. So the area of FJKI, which is known to be 60, can be written as $60 = \frac{1}{2}(6+12)(h)$ where h is the height of trapezoid FJKI which also happens to be FJ. Solving for h yields that $h = \frac{20}{3}$.

C. Looking at FGHI as a trapezoid, notice that the base $FG = 5(\frac{10}{3}) = \frac{50}{3}$. Using this we can see that the area is $\frac{1}{2}(6+21)(\frac{50}{3}) = 225$.

So $A + 3B + C = 12 + 20 + 225 = 257$

5. Plugging our values into Stewart's theorem, we get $\left(4^2\right)(6) + \left(5^2\right)(2) = d^2(8) + 2(6)(8)$. Therefore, $d = \frac{5}{2}$.

6. A: $|30(2) - 5.5(10)| = 5°$

B: The time difference between the two times is 10 hours and 10 minutes. The minute hand travels $360°$ each hour. So the number of degrees it travels in 10 hours and 10 minutes is

$$10(360) + \frac{10}{60}(360) = 3660.$$

C: The tip of the hand will travel around a full circle $10\frac{1}{6}$ times. The distance traveled around the circle once is $2\pi(4) = 8\pi$. So the total distance traveled will be

$$10\frac{1}{6}(8\pi) = \frac{244\pi}{3}.$$

So $\frac{AC}{\pi B} = \frac{5(\frac{244\pi}{3})}{3660\pi} = \frac{1}{9}$.

7. A: By SAS~ ABC~DEF. So assign a value of 1

B. Not enough information to assume anything. Assign a value of 0.

C. By properties of kites, MN=NO and MP=OP. Also MO=MO. Because of all of this we cannot prove that the triangles are similar or congruent. So assign a value of 0.

D: By SAS~ the triangles will be similar. So assign a value of 1.

E. We know that WY=XY by properties of the perpendicular bisector, WA=AX since A is the midpoint of WX and AY=AY by the reflexive property. So by SAS congruence, WYA is congruent to XYA. Since it is congruent it must also be similar. Assign a value of 3.

F. There is not enough information to assume anything. Assign a value of 0.

The sum of the values is 1+0+0+1+3+0=5.

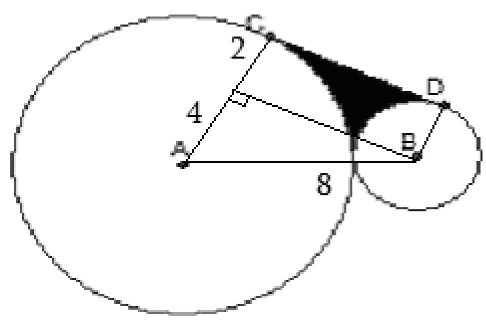
8. The overlapping areas of the circle is $2\left(\frac{120}{360}(4\pi) - \frac{1}{2}\left(2\sqrt{3}\right)(1)\right) = \frac{8\pi}{3} - 2\sqrt{3}$. Notice that AC, CB, BD, DA, and AB all have length 2. Thus, quadrilateral ACBD is a rhombus with one diagonal of 2 and the other of length $2\sqrt{3}$. Therefore, the area inside the overlap but outside the rhombus is $\frac{8\pi}{3} - 2\sqrt{3} - \frac{1}{2}(2)(2\sqrt{3}) = \frac{8\pi}{3} - 4\sqrt{3}$. The line segment connecting the center of the smaller circle to

the side of the rhombus is the radius and the altitude to the hypotenuse of the right triangle formed by one side of the rhombus and half the length of each diagonal. Thus, the radius has length $\frac{\sqrt{3}}{2}$ and the area of the triangle is $\frac{9\sqrt{3}}{16}$. Thus, the area of the shaded region is $\frac{8\pi}{3}-\frac{55\sqrt{3}}{16}$.

9. The slope of AB is $\frac{1-(-2)}{5-2}=1$. So the slope of the line perpendicular to it is -1. Also, the midpoint of AB is $(\frac{2+5}{2},\frac{1-2}{2})=(\frac{7}{2},\frac{-1}{2})$. So the line that is the perpendicular bisector of AB is $y-(\frac{-1}{2})=-1(x-\frac{7}{2})\Rightarrow y=-x+3$. Point C is the y-intercept of this line, so C is (0,3). Let (x,y) be point D.

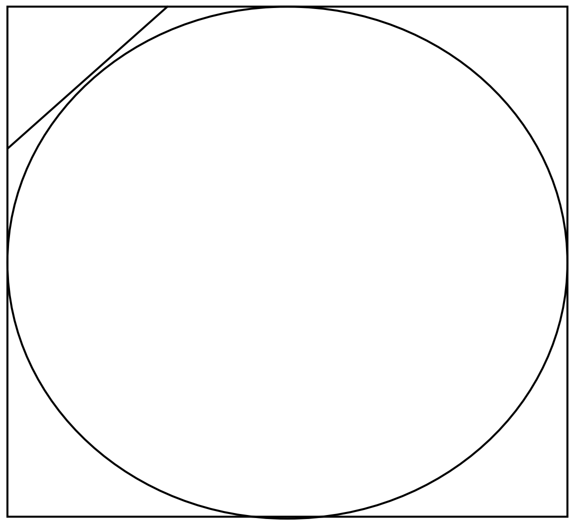
We know that $(\frac{7}{2},\frac{-1}{2})$ is also the midpoint of CD. So $\frac{0+x}{2}=\frac{7}{2}\Rightarrow x=7$ and $\frac{3+y}{2}=\frac{-1}{2}\Rightarrow y=-4$. So the sum of the coordinates of D is 3.

10. A figure is given below:



CD is equivalent to the second leg of a right triangle with first leg of length 4 and hypotenuse of length 8, which makes the length of the external tangent segment $4\sqrt{3}$. The area of the shaded region is equivalent to the area of quadrilateral ABCD minus the area of segment A with radial angle of 60 degrees and the area of segment B with radial angle of 30 degrees. The area of quadrilateral ABCD is $A=(2)\left(4\sqrt{3}\right)+\frac{1}{2}(4)\left(4\sqrt{3}\right)=16\sqrt{3}.$ The area of segment A is $A=\frac{60}{360}(6)^2\pi=6\pi$. The area of segment B is $A=\frac{120}{360}(2)^2\pi=\frac{4\pi}{3}$. So, the area of the shaded region is $A=16\sqrt{3}-6\pi-\frac{4\pi}{3}=16\sqrt{3}-\frac{22\pi}{3}$.

11. First inscribe a circle inside a square. Then, since the circle has a radius of R, the square has a side length of 2R. We can make an octagon using the square as such.



One side is shown in the above diagram. This can be done by cutting the sides of the square into three equal parts and then connecting consecutive marks (as seen in one part above). Notice that this forms 4 right triangles. If the length of the legs of the triangles is "x," the length of the side of the octagon is both $2R-2x$ and $x\sqrt{2}$. Setting these two values equal, we see that $x=2R-R\sqrt{2}$. The sum of the areas of these 4 triangles is $4\left(\frac{1}{2}\right)\left(2R-R\sqrt{2}\right)^2=12R^2-8R^2\sqrt{2}$. Notice that the area of the octagon can be found by subtracting the areas of the triangles from the area of the square. So the area of the octagon is $(2R)^2-(12R^2-8R^2\sqrt{2})=8R^2\sqrt{2}-8R^2$

12. The duck rope should be at the circumcenter of the circle. So the extraneous grazing area is equal to $A_{CircumscribedCircle}-A_{Triangle}$. The lengths of the sides of the triangle are $\sqrt{(4-0)^2+(0-3)^2}=5$, $\sqrt{(0+5)^2+(3+9)^2}=13$, $\sqrt{(4+5)^2+(0+9)^2}=9\sqrt{2}$. The area of the triangle can be calculated by using the magical formula. Line up the x-values and the y-values, and calculate the following:

$$\frac{\left|\left(x_1y_2+x_2y_3+x_3y_1\right)-\left(x_1y_3+x_2y_1+x_3y_2\right)\right|}{2}$$

0	3
4	0
−5	−9
0	3

(this makes it easier to find x and y. $A_{Triangle}=\frac{\left|(12+0+0)-(0-36-15)\right|}{2}=\frac{63}{2}$

The radius of the circumscribed circle is equivalent to $\frac{abc}{4A_{Triangle}}$

$$r=\frac{5(13)(9\sqrt{2})}{4\left(\frac{63}{2}\right)}=\frac{5(13)(9\sqrt{2})}{126}=\frac{65\sqrt{2}}{14},\ A=\pi\left(\frac{65\sqrt{2}}{14}\right)^2-\frac{63}{2}=\frac{65^2\pi}{7(14)}-\frac{63}{2},$$

so $65-63+\frac{7(14)}{2}=2+49=51$.

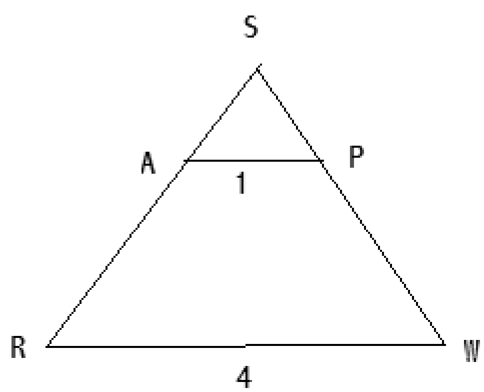
13.The first statement is FALSE because any point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment.

The second statement is FALSE because the lines could be skew.

The third statement is TRUE (for example, a triangle with degree measures of 1,1,178.
The fourth statement is TRUE.
The fifth statement is FALSE. The angle bisector is equidistant from the sides of the angle, but the lengths of BA and BC may not be the same.
The sixth statement is TRUE.
The seventh statement is FALSE since a line cannot have a bisector of any sort since a line is infinite.

Thus, there are 3 true statements.

14. There are 13 books in <u>Elements</u>. So the number of degree measures in an interior angle of a regular 13-Gon is $\frac{180(13-2)}{13}=\frac{1980}{13}$



15.

The height of the triangle is $2\sqrt{3}$. Using proportions, we see that the height, h, of triangle ASP is $\frac{h}{2\sqrt{3}}=\frac{1}{4}\rightarrow h=\frac{\sqrt{3}}{2}$. Therefore, the shortest distance is $2\sqrt{3}-\frac{\sqrt{3}}{2}=\frac{3\sqrt{3}}{2}$.