CALCULUS INDIVIDUAL

For all questions, "E. NOTA" means none of the above answers is correct. Unless otherwise stated, assume all numbers are real.

1. What is the equation of the line tangent to the graph $f(x)=3x^2-40$ when $x=4$?

A. $y=24x-88$ B. $y=4x-12$ C. $y=6x-32$ D. $y=8x-24$ E. NOTA

2. Evaluate: $\lim_{x\to 3}\sqrt{\frac{x^2-5x+6}{x^2-2x-3}}$?

A. $\frac{1}{4}$ B. $\frac{1}{3}$ C. $\frac{2}{3}$ D. 1 E. NOTA

3. What is the area of the region bounded by the graphs $x=y^2-4y+5$ and $y=x-1$.

A. $\frac{1}{4}$ B. $\frac{5}{6}$ C. $\frac{7}{3}$ D. $\frac{9}{2}$ E. NOTA

4. Let $f(x)$ be a monotonic decreasing and concave upward function for all *x*. Let $g(x)$ be a monotonic increasing and concave downward function for all *x*. Suppose $f(x)$ and $g(x)$ have continuous first and second derivatives. Which of the following is true?

A. $y=f(g(x))$ is monotonic increasing and concave upward for all *x*
B. $y=f(g(x))$ is monotonic increasing and concave downward for all *x*
C. $y=f(g(x))$ is monotonic decreasing and concave upward for all *x*
D. $y=f(g(x))$ is monotonic decreasing and concave downward for all *x*
E. NOTA

5. Evaluate: $\int_0^1 \frac{3x^3+5x^2+x}{x+1}\,dx$

A. $1+\ln 2$ B. $3-\ln 2$ C. $9\ln 2$ D. $7+2\ln 2$ E. NOTA

6. $\lim_{x\to\infty}\left(\sqrt{ax^4+bx^3+cx^2+dx+e}-\sqrt{x^4+3x^3-2x^2-4x-7}\right)=c$, where *a*, *b*, *c*, *d*, and *e* are real numbers. What is *c*?

A. 1 B. 2 C. 3 D. 4 E. NOTA

For questions 7 and 8, let $f(x)=\dfrac{x}{1+\dfrac{x}{1+\ldots}}$.

7. Evaluate $f'(6)$.

A. -3 B. $-\frac{1}{5}$ C. $\frac{1}{5}$ D. 2 E. NOTA

8. Evaluate $\int_2^6 f(x)\,dx$

A. $-\frac{61}{6}$ B. $\frac{7}{6}$ C. $\frac{23}{6}$ D. $\frac{37}{6}$ E. NOTA

9. A rectangle initially has a width of two inches and a length of five inches. The width of a rectangle is increasing at a rate of two inches per second while the length of the rectangle is decreasing at a rate of one inch per second. After five seconds the length and width of the rectangle remain constant. What is the maximum area attained by the rectangle? Express your answer in square inches.

A. 2 B. 10 C. 18 D. 20 E. NOTA

10. Let $f(x)$ be a continuous function on $[0,1]$ and differentiable on $(0,1)$ such that the range of $f(x)$ is $[0,1]$. There exists a number c in $[0,1]$ such that $f(c)=c$. This is a consequence of what theorem?

A. Intermediate Value Theorem B. Mean Value Theorem
C. Rolle's Theorem D. Taylor's Theorem E. NOTA

11. Use differentials to approximate $\sqrt[3]{500}$ given that $\sqrt[3]{512}=8$. Round your answer to the nearest thousandth.

A. 7.936 B. 7.937 C. 7.938 D. 7.939 E. NOTA

12. What is the average value of $f(x)=\frac{1}{x}$ on the interval $\left[\frac{2}{3},\frac{8}{3}\right]$?

A. $\ln\sqrt{2}$ B. $\ln 2$ C. 1 D. $\ln 4$ E. NOTA

13. Evaluate: $\sum_{i=0}^{\infty}\frac{(\ln 2)^i}{i!}$

A. $1-\ln 2$ B. 1 C. 2 D. $\frac{1}{1-\ln 2}$ E. NOTA

For questions 14 and 15, let $f(x)=\begin{cases} x^2+ax+b & \text{if } x<1 \\ -x^2+8x+4 & \text{if } x\geq 1 \end{cases}$ where a and b are real numbers.

14. If $f(x)$ is differentiable at $x=1$, then what is $a\cdot b$?

A. 6 B. 12 C. 24 D. 48 E. NOTA

15. If $\int_0^2 f(x)\,dx=20$, then what is $a+2b$?

A. 6 B. 12 C. 24 D. 48 E. NOTA

16. Use a second order MacLaurin series expansion of $f(x)=e^{x^2}+x$ to approximate $\int_0^{1/2}\left(e^{x^2}+x\right)dx$.

A. $\frac{1}{2}$ B. $\frac{2}{3}$ C. $\frac{3}{4}$ D. 1 E. NOTA

17. Let $f(x)=\frac{x^2\sin(x)+3x}{1+\ln\left(x^2+1\right)}$. Find $f'(0)$.

A. 1 B. $\frac{3}{2}$ C. 3 D. 6 E. NOTA

18. Let $\sum_{i=1}^{\infty}|a_i|=N$, where N is a positive integer and each a_i is a real number. Which of the following is true?

A. $\sum_{i=1}^{\infty}(a_i)^2$ converges to a positive integer

B. $\sum_{i=1}^{\infty}(a_i)^2$ converges to a positive number, which may or may not be an integer

C. $\sum_{i=1}^{\infty}(a_i)^2$ diverges

D. It cannot be determined if $\sum_{i=1}^{\infty}(a_i)^2$ converges or diverges

E. NOTA

19. If $\int_{-\infty}^{\infty} \frac{1}{\sqrt{\pi}} e^{-x^2}\,dx = 1$, then what is $\int_{-\infty}^{\infty} e^{-x^2+4x-3}\,dx$?

A. $e^{-3}\sqrt{\pi}$ B. $e\sqrt{\pi}$ C. $e^{3}\sqrt{\pi}$ D. $e^{4}\sqrt{\pi}$ E. NOTA

20. Let $f(x)$ be a function with continuous first derivatives for all real x such that $xf\left(x^2\right) - xf(4) = xf(2x) - 2f(2x)$. If $f(4) = 12$, what is $f'(0)$?

A. 0 B. $\frac{3}{2}$ C. 3 D. 4 E. NOTA

21. $\ln 2$ can be written in the form $A \ln B$ where $B > 1$. What is the minimum value of $A \cdot B$?

A. $\frac{1}{\ln 2}$ B. $e \ln 2$ C. 2 D. e E. NOTA

22. Evaluate: $\frac{d}{dx}\int_{x}^{2x} \ln t^3\,dt$

A. $\ln(2x)$ B. $2\ln(8x)$ C. $3\ln x$ D. $3\ln(4x)$ E. NOTA

23. Let $y = \left[x^{\alpha-1} + 1\right]^{-1}$, where $0 < \alpha < 1$. Which of the following is true when $x = 2$?

A. y is increasing at an increasing rate
B. y is increasing at an decreasing rate
C. y is decreasing at an increasing rate
D. y is decreasing at an decreasing rate
E. NOTA

24. Evaluate: $\lim_{x\to\infty}\left(1 + \frac{3}{x}\right)^{2x}$

A. $e^{\frac{2}{3}}$ B. e C. $e^{\frac{3}{2}}$ D. e^{6} E. NOTA

25. Let x and y be two positive numbers whose sum is ten. What is the smallest possible value of $x^2 - xy + 2y$?

A. 2 B. 3 C. 4 D. 5 E. NOTA

26. Let $f(x) = \left[\log_2(x)\right]^2$. What is $f'(2)$?

A. 1 B. $\log_2(e)$ C. 2 D. e E. NOTA

27. Evaluate: $\lim_{n\to\infty}\sum_{i=0}^{n}\frac{n}{n^2-2in+2i^2}$

A. $\frac{\pi}{4}$ B. $2\arctan\left(\frac{1}{2}\right)$ C. 1 D. $\frac{\pi}{2}$ E. NOTA

28. Jennifer is a widget manufacturer who sells a case of widgets for \$100 apiece. If a customer orders more than 50 cases, Jennifer awards a bulk discount, decreasing the price of each case \$1 times the number of cases ordered in excess of 50. If Jennifer wishes to maximize her revenue, how many cases of widgets should she sell to a single customer?

A. 25 B. 50 C. 75 D. 100 E. NOTA

29. Let $f(x)=x^2e^x$. Evaluate $f^{(10)}(10)$.

A. $100e^{10}$ B. $170e^{10}$ C. $240e^{10}$ D. $390e^{10}$ E. NOTA

30. Evaluate: $\lim_{x\to 4}\frac{x^2+4x}{x+4}$

A. 0 B. 2 C. 8 D. 12 E. NOTA