Capital City Regional **3/8/2008**

PRECALCULUS BOWL SOLUTIONS

1. $9\frac{1}{3}\ or \frac{28}{3}$ **Part A—**We have $a+30d=16\ \&\ a+72d=46.$ Solving gives $a=-2\ \&\ d=\frac{2}{3}.$ Hence, the 6th term is $\frac{4}{3}.$ **Part B—**We have $ar=4\ \&\ ar^5=16.$ Solving gives $r=\sqrt{2}\ \&\ a=2\sqrt{2}.$ Hence, the 4th term is 8. Sum the parts to get the answer.

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2. $\frac{53}{112}$ **Part A—**This must be split into 2 inf. series: $\frac{1}{7}+\frac{1}{7^2}+\frac{1}{7^3}+\cdots=\frac{1}{6}\ \&\ \frac{1}{7^2}+\frac{1}{7^4}+\frac{1}{7^6}+\cdots=\frac{1}{48}.$ The sum of these series is $\frac{3}{16}.$ **Part B--**Split the series again: $1-\frac{1}{2}-\frac{1}{4}-\frac{1}{8}-\square=1-\frac{\frac{1}{2}}{1-\frac{1}{2}}=0$ & $\frac{2}{8}+\frac{2}{64}+\cdots=\frac{\frac{2}{8}}{1-\frac{1}{8}}=\frac{2}{7}.$ The sum of these series is $\frac{2}{7}.$ Sum the parts to get the answer.

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3. 230 **Part A—**For ea question, there are 2 choices. So, there are $2^5=32$ choices. **Part B—**The books that must be together are treated as a single unit. Those books may be in 2 diff orders, so mult by 2 at the end. We have 4 book units that can be arranged in 4! = 24 ways. Mult by 2 =48 ways. **Part C—**A palindrome is of the form tht . The h digit is chosen in 9 ways (can't be 0), and the t digit is chosen in 10 ways. The total # is 90 ways. **Part D—**We need 1 of 4 flavors and 2 of 6 toppings. This is C(4,1) · C(6,2) = (4)(15)=90. Sum the parts for the answer

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4. 52 **Part A—**First, count the 4 corner posts, then 9 posts (side of length 20) to the next corner, then 5 posts (side of width 12), to the next corner, etc, for a total of 32 posts. **Part B—**Ten degrees = $\frac{5}{3}$ mins. So, we need the time when the hour hand is $\frac{5}{3}$ mins from the min. hand. Let the min. hand be at x mins. So, the hour hand is $\frac{x}{60}$ of the way from hour 4 to hour 5. The hour hand is at min $20+5\left(\frac{x}{60}\right)=20+\frac{x}{12}.$ The 1st time the 2 hands are 10 degrees apart, the min hand is before the hour hand. So, $x+\frac{5}{3}=20+\frac{x}{12}.$ This gives x =20. Sum parts for the answer.

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5. 235 **Part A—**There will be no inverse exactly when the determinant = 0. Solving the 3 x 3 determinant gives c = -1. **Part B—**The determinant of a product is the product of the determinants. So, the determinant of D = (2)(4) - (1)(3) = 5, of E is (2)(4) – (3)(5) = -7, and of F is (2)(1) – (1)(6) = -4. The product is 140. **Part C—**Solving this determinant by row reducing and then solving, gives 96. Sum the parts to get the answer.

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6. $\frac{163}{36}$ **Part A—**Use the identity $sin^2x + cos^2x = 1$ to add together all 8 trig terms, paring the terms with identical angles. This sum is 4, and the sum of the squares of the sines is $\frac{9}{4}$, so the required sum is $4 - \frac{9}{4} = \frac{7}{4}$. **Part B—**Since $\tan x = \frac{1}{\cot x}$ and $\cot x = \frac{1}{\tan x}$, then rewrite $\frac{1}{\tan x} + \frac{1}{\cot x}$ as $\cot x + \tan x$, which is given as $\frac{25}{9}$. Sum the parts to get the answer.

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7. $1\frac{1}{8}$ *or* $\frac{9}{8}$ **Part A—**Factoring the bottom gives $(\sqrt{x} - 4)(\sqrt{x} + 4)$. Cancelling leaves the problem as $\frac{1}{\sqrt{x} + 4}$, and the limit is $\frac{1}{8}$. **Part B—**The denom. goes to infinity as the num. oscillates btwn 1 & -1, so the limit is 0. **Part C—**Mult the top and bottom by the conjugate of the bottom and simplify to get $\frac{4x}{\sqrt{4x^2 + 5x} + \sqrt{4x^2 + x}} = \frac{4}{\sqrt{4 + \frac{5}{x}} + \sqrt{4 + \frac{1}{x}}} = \frac{4}{4} = 1.$ Sum the parts to get the answer.

8. $78\frac{1}{2}$ *or* $\frac{157}{2}$ **Part A—**We get (2 + 4 + 6 +...+ 38 + 40) – (1 + 3 +5 +...+37 + 39) = 1 + 1 + 1 +...+1 + 1 = 20. **Part B—**The avg of the 1st 50 positive integers is the same as the avg of the 1st and last of these integers, hence, 25.5. **Part C—**We know the sum of n integers is $\frac{1}{2}n(n + 1)$. We want $\frac{1}{2}n(n + 1) \le 200.$ The largest such n is 19. **Part D—**If term 4 + term 5 = term 6, then term 5 = term 6 – term 4 = 6 – 4 = 2. Then term 7 = term 5 + term 6 = 2 + 6 = 8. So, term 8 = term 6 + term 7 = 6 + 8 = 14. Sum the parts to get the answer.

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9. **4** **Part A—**To find $y = f^{-1}(-2)$, we write $-2 = f(y) = \frac{1}{y + 2} \rightarrow y = -\frac{5}{2}$. Then $g(f^{-1}(-2)) = g\left(-\frac{5}{2}\right) = 3$. **Part B—**to find $f\left(\frac{1}{2}\right)$, we need to find x such that $g(x) = 1 - x^2 = \frac{1}{2} \rightarrow x = \frac{1}{\sqrt{2}}$. So, $f\left(\frac{1}{2}\right) = f\left(g\left(\frac{1}{\sqrt{2}}\right)\right) = 1.$ Sum the parts to get the answer.

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10. **2** **Part A—**Both the numerator's and the denominator's values are 1 because the value of ea of the products (sin) (csc) and (cos)(sec) and (tan)(cot) is 1. **Part B—**Since $\tan x = \frac{1}{\tan(90 - x)}$, then $\tan 15 \cdot \tan 75 = 1$ & $\tan 30 \cdot \tan 60 = 1$. So, (1)(1)$\tan 45 = 1$. Sum the parts to get the answer.

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11. $33\frac{3}{4}$ or $\frac{135}{4}$ **Part A—**The reciprocals of the sequence are $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \ldots$ This is now an arithmetic sequence and the 8th term is $\frac{4}{3}$. The 8th term in the harmonic sequence is the reciprocal of this, or $\frac{3}{4}$. **Part B—**Using the given relation, we get $a_3 = 33.$ Sum the parts to get the answer.

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12. **554** **Part A—**Convert from log form to exponential form: $\cos x = \frac{1}{\sqrt{2}} \to x = 45.$ **Part B—** Converting again gives $b^2 = \log_3 x^2 \to (\log_3 x)^2.$ Rearrange & factor to get $(\log_3 x)(-2 + \log_3 x) = 0 \to \log_3 x = 2 \;\&\; x = 9.$

Part C— $(\log_x 2x)(\log_{10} x) = \left(\frac{\log 2x}{\log x}\right)\left(\frac{\log x}{\log 10}\right) = \log_{10} 2x = 3 \to 10^3 = 2x \to x = 500.$ Sum the parts to get the answer.

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13. $\frac{1}{26} + \frac{16}{25}$ **Part A—**This is simply a counting problem. There are 52! ways to order the cards. To count the ways that 2 given cards can be adjacent, we consider them as a unit, so we can now order 51 cards in 51! ways. Since the 2 given cards can be ordered in 2 ways, the final probability is [2(51!)]/52! = 1/26. **Part B—**Applying P(A|B) = $\frac{P(A \cap B)}{P(B)}$, we get

P = $\frac{\left(\frac{1}{2}\right)\left(\frac{4}{6}\right)}{\left[\left(\frac{1}{2}\right)\left(\frac{4}{6}\right) + \left(\frac{1}{2}\right)\left(\frac{3}{8}\right)\right]} = \frac{16}{25}.$ Sum the parts to get the answer.

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14. **119** **Part A—**The stamps which are not on the border form an 8 x 8 square of 64 stamps. Thus, the required probability is 64% (we use 64). **Part B—**The largest box is not empty; it contains 10 smaller boxes. Since exactly 6 boxes are not empty, then 5 of the 10 smaller boxes are empty. The other 5 smaller boxes each contain 10 empty boxes. Therefore, the number of empty boxes is 5 + (5)(10) = 55. Sum the parts to get the answer.
