PRECALCULUS INDIVIDUAL SOLUTIONS

1. C-- $(x+y)^2 = x^2 + y^2$ simplifies to $xy = 0$. This is the eqn of the coordinate axes (2 lines).

2. D—A parabola with the given info must have equation $y = a(x-p)^2 + p$. B/c the y-intercept is $(0, -p) and\ p \neq 0$, then $a = -\frac{2}{p}$. So, $y = -\frac{2}{p}(x^2 - 2px + p^2) + p = -\frac{2}{p}x^2 + 4x - p$, so $b = 4$.

3. B—There are only 2 possible occupants for the driver's seat. After the driver is chosen, any of the remaining 3 people can sit in the front, and there are 2 arrangements for the other 2 people in the back. So, there are $2 \cdot 3 \cdot 2 = 12$ possibilities.

4. A—Simplifying, we have $\sqrt{\frac{x}{1-\frac{x-1}{x}}} = \sqrt{\frac{x}{\frac{x-x+1}{x}}} = \sqrt{\frac{x}{1/x}} = \sqrt{x^2} = |x|$. When $x < 0$, then the expression is equivalent to $-x$.

5. D—A puts a ball into B's bag, his bag now has 6 balls. If B puts 1 in A's bag, the 2 bags will have the same contents if and only if B picked one of the 2 balls in his bag that are the same color. B/c there are 6 balls in his bag when he picks, the probability of picking 1 of the same colored pair is 1/3.

6. A—Using $\tan\theta = \frac{\sin\theta}{\cos\theta} \to \frac{2\sqrt{3}}{3}k = \frac{k}{\cos\theta} \to \cos\theta = \frac{\sqrt{3}}{2} \to \theta = \frac{\pi}{6}$

7. C—If $2a$ & $2b$ are the lengths of the major and minor axes and the dimensions of the rectangle are x & y. Then $x+y$ is the sum of the distances from the foci to the pt A on the ellipse, which is $2a$. The length of the diag. of the rect. is the distance btwn the foci of the ellipse, $2\sqrt{a^2-b^2}$. So, $x^2+y^2 = 4a^2 - 4b^2$. The area of the rectangle is $2008 = xy = \frac{1}{2}[((x+y))^2 - ((x)^2+y^2)] = \frac{1}{2[(2(a))^2 - (4a^2-4b^2)]} = 2b^2 \to b = \sqrt{1004}$. So, the area of the ellipse is $2008\pi = \pi ab = \pi a\sqrt{1004} \to a = 2\sqrt{1004}$. the perimeter of the rectangle is $2(x+y) = 4a = 8\sqrt{1004} = 16\sqrt{251}$.

8. B—The period of the function is $\frac{6\pi}{25}$ & the shift is $\frac{\pi}{50}$ right. There are $8\frac{1}{3}$ periods in the given interval, & the graph of ea period has 2 zeros (incl. one at the start). This gives 17 zeros in all.

9. A—The items cost approximately 8 + 5 + 3 + 2 + 1 = 19. So, her change is about $1, which is 5% of her $20.

PRECALCULUS INDIVIDUAL SOLUTIONS

10. A—In order to catch up to John, Bob must walk 1 mile farther in the same amount of time. B/c B0b's speed exceeds John's speed by 2 mph, the time required for Bob to catch up is 30 min.

---

11. D—The conditions on f imply that both $x = f(x) + f\left(\frac{1}{x}\right)$ & $\frac{1}{x} = f\left(\frac{1}{x}\right) + f\left(\frac{1}{\frac{1}{x}}\right) = f\left(\frac{1}{x}\right) + f(x).$ So, if x is in the domain of f, $x = \frac{1}{x} \rightarrow x = \pm 1.$ The conditions are satisfied if and only if $f(1) = \frac{1}{2}$ & $f(-1) = -\frac{1}{2}.$

---

12. A-- $\left(\frac{1}{1-x} - \frac{3}{1-x^3}\right) = \frac{x^2 + x - 2}{(1-x)(1+x+x^2)} = \lim_{x \to 1}\left(\frac{-(x+2)}{x^2+x+1}\right) = -1.$

---

13. B-- $\sum_{n=0}^{\infty} \frac{1}{n^2 + 5n + 6} = \sum_{n=0}^{\infty} \frac{1}{n+2} - \sum_{n=0}^{\infty} \frac{1}{n+3} = \left[\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots\right] - \left[\frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots\right] = \frac{1}{2}$

---

14. D—Note the 1st several terms of the sequence are: $\frac{\frac{2, \frac{3,3}{2,1}}{2,1}}{\frac{3,2}{3}}, 2, 3, \ldots$ so the sequence consists of a repeating cycle of 6 terms. Since 2008 has remainder 4 when divided by 6, $a_{2008} = a_4 = \frac{1}{2}.$

---

15. C— $\frac{C(3,2) \cdot C(5,2)}{C(8,4)} + \frac{C(3,3) \cdot C(5,1)}{C(8,4)} = \frac{3 \cdot 10 + 1 \cdot 5}{70} = \frac{1}{2}.$

---

16. C—Let the 1st odd integer $= x$. Then $x + (x+2) + (x+4) + \cdots + (x+198) = 100^{100}.$ Or, $100x + 2 + 4 + 6 + \cdots + 198 = 100x + 2(1 + 2 + 3 + \cdots + 99) = 100x + 99 \cdot 100 = 100^{100}.$ Then, $x + 99 = 100^{99} \rightarrow x = 100^{99} - 99.$

---

17. A—Using abs. value for area, we get $|ABC| = |OAC| - |OAB| - |OBC|,$ where O = (0, 0). $|OAC| = \frac{1}{2} 6 \cdot 4 \sin(AOC) = 12 \sin 120^{\circ} = 6\sqrt{3}$ and $|OAB| = 3\sqrt{3}$ and $|OBC| = 2\sqrt{3},$ all by the same method. So, $|ABC| = 6\sqrt{3} - 5\sqrt{3} = \sqrt{3}.$

---

18. B--
$[2(4]^x) + 6^x = 9^x \rightarrow 2^{2x+1} + 2^x 3^x - 3^{2x} = 0 \rightarrow ($
$2^{\uparrow}(x+1) - 3^{\uparrow}x)(2^{\uparrow}x + 3^{\uparrow}x) = 0 \rightarrow 2^{\uparrow}(x+1) = 3^{\uparrow}x \rightarrow (\square 2/3)\square^{\uparrow}x = 1/2.$ This means that $x = log_{\frac{2}{3}} \frac{1}{2}$ & $a = \frac{1}{2}.$

PRECALCULUS INDIVIDUAL SOLUTIONS

19. B—$y^2 - 4xy + 4x^2 - 2x + y - 12 = 0 \to (y-2x)^2 + (y+2x) - 12 = 0 \to$
$(y-2x+4)(y-2x-3) = 0 \to y - 2x = -4 \;\&\; y - 2x = 3.$ These are 2 parallel lines.

---

20. C—S= $\frac{1}{5} + \frac{2}{5^2} + \frac{3}{5^3} + \cdots \& \frac{1}{5}S = \frac{1}{5^2} + \frac{2}{5^3} + \cdots$ Subt. gives $\frac{4}{5}S = \frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} + \cdots = \frac{\frac{1}{5}}{1 - \frac{1}{5}} = \frac{1}{4}.$ So, S= $\frac{5}{16}$.

---

21. C—We find this by using $\frac{12!}{6!4!2!} = 13860.$

---

22. B—Completing the square gives $(x-10)^2 + (y-12)^2 = 4 \to r = 2$. The distance from the center of the circle to the given point is $\sqrt{113}$. Hence, the distance required is $\sqrt{113} - 2.$

---

23. C—Using the vertical component to be 18 and the horizontal component to be $12\pi,$ then the distance traveled is $\sqrt{18^2 + (12\pi)^2} = 6\sqrt{(9 + 4\pi^2)}.$

---

24. A--$\frac{1}{0}$ is not an indeterminate form, it is either undefined or infinity.

---

25. A—Plugging in 15, you get $15^3 - 7(15)^2 - \frac{1}{3}a(15) - 9b = 1800 - 5a - 9b.$ The least possible solution is (0, 200) and their product is 0.

---

26. E (12)—One can either foil and simplify, then plug in), or one can recognize the function as x^3 and the derivative as 3$x^2 @ x = 2 \to 12.$

---

27. D—$33\frac{1}{3}\ rpm = \frac{\frac{100}{3}\ rev}{1\ min} = \frac{\frac{100}{3}(2\pi) rads}{1\ min} = \frac{200\pi}{3}.$

---

28. E (All)—$(1)^{\uparrow}(1/4) = (cis\ 0^{\uparrow o})^{\uparrow}(1/4) = cis\ 0^{\uparrow o}, cis\ \square 90 \square^{\uparrow o}, cis\ \square 180 \square^{\uparrow o}, cis\ \square 270 \square^{\uparrow o}.$ Hence, all are roots of 1.

---

29. B—Consider $f(x) = (x-2)(x-7),$ then $f\left(\frac{x}{2}\right) = \left(\frac{x}{2} - 2\right)\left(\frac{x}{2} - 7\right)$, which has roots at 4 & 14.

---

30. D--$a_1 = 24 \;\&\; a_{13} = 15$ So $15 = 24 + (13-1)d \to d = -\frac{3}{4}$ The rungs are then: $24, 23\frac{1}{4},$ $22\frac{1}{2}, 21\frac{3}{4}, 21, 20\frac{1}{4}, 19\frac{1}{2}, 18\frac{3}{4}, 18, 17\frac{3}{4}, 16\frac{1}{2}, 15\frac{3}{4}, and\ 15.$ The only match is 18.