GEOMETRY BOWL SOLUTIONS

T(1) Part A:

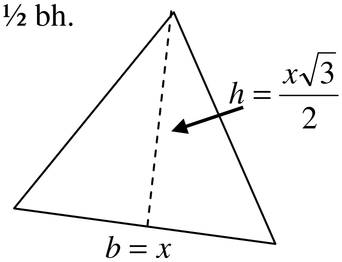
Use the formula: sum of angles = $180(n-2)$

$7380 = 180(n-2)$ $\quad 7740 = 180n$

$7380 = 180n - 360$ $\quad n = 43$

Part B:

Because of equilateral triangle, each angle is 60 degrees. Area = ½ bh.



$81\sqrt{3} = \frac{1}{2}bh$

$81\sqrt{3} = \frac{1}{2}(x)\left(\frac{x\sqrt{3}}{2}\right)$

$81\sqrt{3} = \frac{x^2\sqrt{3}}{4}$

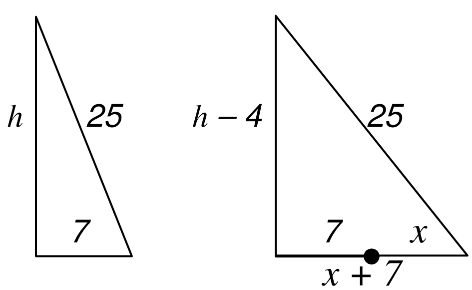
$324\sqrt{3} = x^2\sqrt{3}$

$324 = x^2 \Rightarrow x = 18 \Rightarrow P = 54$

43 x 54 = **2322**

T(2) If there are 100 people around the table each taking 2 ft along the edge, circumference is 200 ft. $C = \pi d \quad 200 = (3.14)d$ d ≈ **63.69**

T(3)



Using Pythagorean Theorem,

$7^2 + h^2 = 25^2 \Rightarrow h = 24$

$h - 4 = 20$

$20^2 + (x+7)^2 = 25^2$

$(x+7)^2 = 625 - 400 = 225$

$x + 7 = 15$

$x =$ **8**

T(4) Part A: $V = \frac{1}{3}BH$

$3168 = \frac{1}{3}\left[\frac{1}{2}(20+28)h\right](36)$

$3168 = \frac{1}{3}(24)(36)h$

$3168 = 288h \Rightarrow h = 11$

Part B:

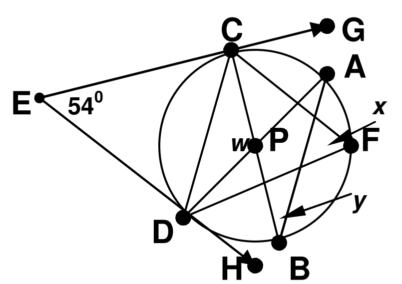
$V = lwh$

$V = 5 \bullet 6 \bullet \frac{10}{12} = 25$

$25 \bullet 63 = 1575$ pounds

1575 + 11 = **1586**

T(5)



$w = \angle CPD$

$x = \angle DFC$

$y = \angle ABC$

The tangents ED and EC form an isosceles triangle EDC, thus $\angle CDE = \angle DCE = 63^0$ each. Triangle PCD is also isosceles because all radii are congruent and tangents are perpendicular to the radii at C and D. $\angle ECB = 90^0$, subtract $\angle DCE = 27^0$.

$w + \angle PCD + \angle PDC = 180^0$. Therefore, w = 126^0 and $\angle APB = 126^0$, making y = 27^0.

$\angle E = \frac{1}{2}(\text{major arc } CFD - \text{minor arc CD})$

Let z =minor arc CD, then 360 – z = major arc CFD

$54^0 = \frac{1}{2}[(360 - z) - z]$

$z = 126^0$, the degree measure of arc CD. Angle CFD is an inscribed angle = ½ arc CD or x = 63^0.

$2w + 3x - 5y = 2(126) + 3(63) - 5(27) = 306$

T(6)

AB = 40 $\quad$ AC = CB = 20

AD = DC = 10 $\quad$ DE = EC = 5

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EC + CB = EB = 5 + 20 = 25

$25^2 =$ **625**

T(7) 1^{st} has 9 circles; 2^{nd} has 12 circles; 3^{rd} has 15 circles; pattern is a difference of 3.

$a_{50} = 9 + 3(50-1) = 9 + 3 \bullet 49 = \mathbf{156}$

T(8) A = False B = False C = True
D = False E = False F = True
G = False H = True

E	H	B	D	F	G	C	A	
0	1	0	0	1	0	1	0	= 1,001,010

T(9) The circumcenter point, or center of the circle, is created by the intersection of the perpendicular bisectors. First find the midpoints of each side of the triangle created by joining the points A(-14, 4), B(3, 11), and C(11, -1).

Midpoint AB: $\left(\frac{-14+3}{2}, \frac{4+11}{2}\right) = \left(\frac{-11}{2}, \frac{15}{2}\right)$

Midpoint BC: $\left(\frac{3+11}{2}, \frac{11+(-1)}{2}\right) = (7,5)$

Midpoint AC: $\left(\frac{-14+11}{2}, \frac{4+(-1)}{2}\right) = \left(\frac{-3}{2}, \frac{3}{2}\right)$

And then find the slopes of the 3 sides of the triangle in order to find the slopes of the perpendicular bisecting lines.

Slope AB: $\frac{4-11}{-14-3} = \frac{7}{17}$ $\perp slope = \frac{-17}{7}$

Slope BC: $\frac{11-(-1)}{3-11} = \frac{3}{-2}$ $\perp slope = \frac{2}{3}$

Slope AC: $\frac{4-(-1)}{-14-11} = \frac{1}{-5}$ $\perp slope = 5$

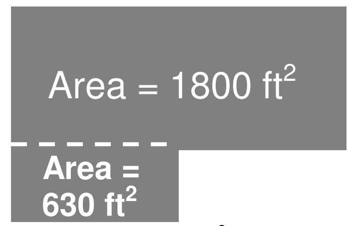
The equation of the line through the midpoint of AB with $slope = \frac{-17}{7}$ is $17x + 7y = -41$.

The equation of the line through the midpoint of BC with $slope = \frac{2}{3}$ is $2x + 3y = -1$.

The equation of the line through the midpoint of AB with $slope = 5$ is $5x - y = -9$.

Solving these system of equations gives an intersection point of **(-2, -1)**

T(10)



Total area = 2430 ft^2

Volume = 2430 x $\frac{4}{12} = 810$ ft^3

1 cubic yard = 27 cubic feet

$810 \div 27 = 30$

$30 \bullet \$32 \bullet 10$ slabs = **$9,600.**

T(11) Formula for area is $A = \frac{1}{2}aP$, $a = 4\sqrt{3}$

Large hexagon area = $\frac{1}{2}(2 \bullet 4\sqrt{3})(6 \bullet 16) = 384\sqrt{3}$

Small hexagon area = $\frac{1}{2}(4\sqrt{3})(6 \bullet 8) = 96\sqrt{3}$

$384\sqrt{3} - 96\sqrt{3} = 288\sqrt{3}$ cm^2

T(12) Part A: $\frac{3}{7} = \frac{a}{8} \Rightarrow a = \frac{24}{7}$

$\frac{4}{7} = \frac{b}{8} \Rightarrow b = \frac{32}{7}$

Part B: Ratio of volumes: $\frac{9^3}{4^3} = \frac{729}{64}$

Part C: 4 walls x 3 m x 3 m = 36 m^2

$$\frac{\frac{b}{a} \bullet c}{d} = \frac{\frac{32}{7} \div \frac{24}{7} \bullet \frac{729}{64}}{36} = \frac{27}{64}$$

Capital City Regional **3/8/2008**

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T(13) There are 80 shaded and 136 unshaded squares for a total of 216 squares.

Unshaded:shaded = $\frac{136}{80} = \frac{17}{10}$

T(14) Pattern is $\frac{n(n-3)}{2} = 560$

$n^2 - 3n = 1120$

$n^2 - 3n - 1120 = 0$

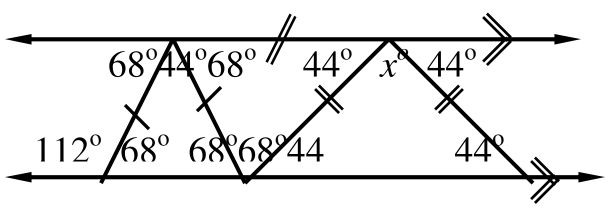
$(n-35)(n+32) = 0$

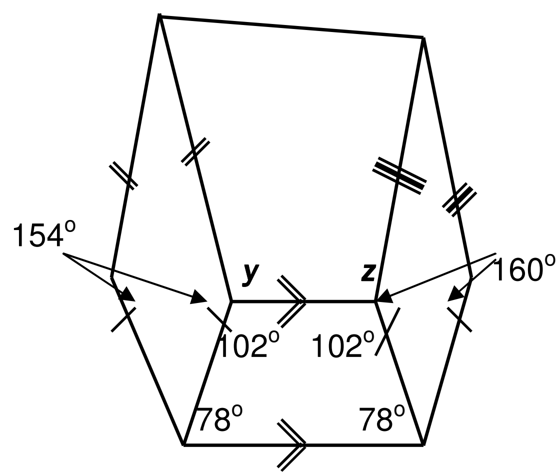
$n = \{35\}$

Therefore polygon has 35 sides and therefore, **35** vertices.

T(15)

Part A: x = $\mathbf{92}^{o}$





Part B: y = 360 – (154 + 102) = 104
z = 360 – (160 + 102) = 98

2(104) + 3(98) – 4(92) = 134