1. **14**—$r(x) = x^3 - 6x^2 + 11x - 6 = (x-1)(x-2)(x-3); 1^2 + 2^2 + 3^2 = 14$
2. **1**—A = 8/2 = 4; B = $y(2) = 2(4) - 8(2) + 5 = -3$; A + B = 1
3. **7, 7th, or February 7**— $y^2 - 10y + 9 = 0 \rightarrow (y-1)(y-9) = 0;$ since y must be at least 3, only $y = 9$ is a possibility. February corresponds to 2, so 9 – 2 = 7.
4. $\sqrt{2}/2 - 2/3$ or $(3\sqrt{2} - 4)/6$ — $3xy = x^2 + x \rightarrow 3y = x + 1 \rightarrow y = \frac{1}{3}(x+1)$ → A, B = 1/3, -1. $x\sqrt{1-x^2} = \dfrac{1}{2} \rightarrow x^2 - x^4 = \dfrac{1}{4} \rightarrow 4x^4 - 4x^2 + 1 = 0 \rightarrow (2x^2 - 1)^2 = 0; 2x^2 = 1 \rightarrow x = \pm\sqrt{2}/2$; only the positive solution works. D has no solutions; since $\sqrt{1-x^2} \geq 0, |x| \leq 1$; there is no point at which one of these functions takes on a value greater than 1, or at which both take on a value of 1, and hence there are no solutions.
5. **144**—From the first statement, it can be seen that a breaded integer must be a multiple of LCM(1, 2, 3, 4) = 12. From the fourth, it can be seen that the first few possibilities for a breaded integer are 36, 144, and 216. Testing 36, we see that it has 9 factors; this does not satisfy the last requirement. Since 144 is a multiple of 36, clearly it must have more factors than 36; hence, the smallest breaded integer is 144.
6. **39/2**—$(x-y)^2 = x^2 - 2xy + y^2 = 4$; since $x^2 + y^2 = 5, -2xy = 1$ and thus A = -½; since $f(x)$ has no x^3 term, B = 0; for part C, clearly (a, b) > 5; the only solutions are (6, 30) and (10, 10); 10 + 10 = 20. 20 – ½ = 39/2.
7. **0**—Since (0, 3) is on the graph, clearly C = 3. Plugging in the other two points: 4A + 2B + 3 = 1 → 2A + B = -1; 25A + 5B + 3 = 28 → 5A + B = 5. Solving these two equations, we get A = 2 and B = -5; hence, 2 – 5 + 3 = 0.
8. **-84**—$2^x = x^2$ when $x = 2$ or $x = 4$ → A = 6; the polynomial in B factors into $(x-41)(x+2) = 0$, with solutions 41 and -2 → B = -2; C = 5(3) – 5 – 3 = 7; ABC = -84.
9. **100**—A = 12; B = $\frac{1}{2}(8)\left(\sqrt{17^2 - 8^2}\right) = \frac{1}{2}(8)(15) = 60$; C = 2 + 3 + 5 + 7 + 11 = 28; 12 + 60 + 28 = 100.
10. **23/2**—$y = k(x-13)(x-37)$; plugging in (25, 36), we get k = -¼, and thus a downward-shaping parabola; for part B, $\frac{-1}{4}(x-13)(x-37) = \frac{-1}{4}\left(x^2 - 50x + 13 \cdot 37\right)$; N = 25/2; -1 + 25/2 = 23/2.
11. **65**—Let y represent my current age and b my brother's current age; then, $(y-10) = \frac{1}{2}(y+4) \rightarrow y = 24$. Ten years ago, my brother was half as old as I was, so he was 14/2 = 7 years old; thus, b = 7 + 10 = 17. When I am 3 times as old as I am now, I'll be 72; it's a difference of 48 years, and so my brother will be 17 + 48 = 65.
12. **32**—The area in question is a diamond, which can be thought of as four congruent right triangles (since the area in each quadrant is the same) of base 4 and height 4; hence, 4(½)(4)(4) = 32.
13. **60/11**—Each minute, I complete 1/30 of the job, Svetlana completes 1/10 of the job, and Chad completes 1/20 of the job; hence, in total (1/30 + 1/20 + 1/10) = (11/60) of the job is being completed, so it'll take 1/(11/60) = 60/11 minutes to rob the bank.
14. **27**—Since (0, 20) is on the curve, C = 20; plugging in (1, 27), we get A + B = 7, so A + B + C = 27.
15. **224**—Call this "certain number" k, call the "second number" m, and call the third number n. $\sqrt{k} = 3\sqrt[3]{m}$ and $\sqrt[3]{m} = 2\sqrt[4]{n}$. Suppose m = 8; this would make k = 36, but n = 1, which does not work. Trying the next even cube, let m = 64; this would set k = 144, and n = 16. All of these are even; 64 + 144 + 16 = 224.