Sequences and Series (Calc Division)
Mu Alpha Theta Convention 2000

Answer choice E. NOTA indicates 'None of the above answers are correct'.

1. For n=2, 3, 4,... a sequence is formed as the positive difference of n^2 and $(n-1)^2$. What type of sequence is formed and what is its common difference/ratio?

A. arithmetic; 2 B. geometric; 2
C. arithmetic; 3 D. geometric; 3 E. NOTA

2. The sequence 1, 2, 5, 6 is repeated 20 more times to form a sequence with a total of 84 numbers. What is the sum of all the terms of this sequence?

A. 280 B. 294 C. 1176 D. 1680 E. NOTA

3. In an arithmetic sequence of 16 terms, the common difference is 2 and the terms' sum is 2,000. What is the sequence's 10th term?

A. 65.5 B. 67.5 C. 128 D. 130 E. NOTA

4. A ball is dropped from 2,000 ft above the ground. On each successive bounce, the ball reaches 1/6 of the previous bounce's height. In all, how many feet does the ball travel? (Assume infinitely many bounces.)

A. 2,400 B. 2,800 C. 4,800 D. 6,800 E. NOTA

5. If the first two terms of the Fibonacci sequence are both 1, what is the absolute value of the difference of the 9th and 10th terms?

A. 13 B. 21 C. 34 D. 55 E. NOTA

6. $(-20)^2+(-19)^2+(-18)^2+\ldots+18^2+19^2+20^2=?$

A. 0 B. 420 C. 2870 D. 5740 E. NOTA

7. Find the geometric mean of 50 and 98.

A. 70 B. 74 C. 4900 D. 5476 E. NOTA

8. In a geometric sequence of 4 terms, the common ratio is -3 and the terms' sum is -100. What is the final term of this sequence?

A. -5400 B. -135 C. 405 D. 2700 E. NOTA

9. Find the infinite sum: $\frac{10}{2}+\frac{7}{4}+\frac{4}{8}+\ldots+\frac{13-3n}{2^n}+\ldots$

A. 3 B. 7 C. 9.25 D. diverges E. NOTA

10. Determine the value of q: $\sum_{i=1}^{20}\begin{vmatrix} 3i & 13 \\ 2 & q \end{vmatrix}=2000.$

A. 2 B. 3 C. 4 D. 5 E. NOTA

11. The $\left(\frac{3n+1}{4}\right)$ th term of a sequence is n^2-5n+2 for $n=1,\frac{7}{3},\frac{11}{3},\ldots.$ What is the sequence's 10th term?

A. Doesn't exist B. $\frac{29}{4}$ C. 52 D. 106 E. NOTA

12. A square of side 4cm is drawn. A second square is drawn with vertices at the midpoints of the first square. If this pattern is continued infinitely, what's the sum of the perimeters of 'all' the squares (in cm)?

A. 4 B. 32 C. $16\sqrt{2}$ D. $64+32\sqrt{2}$ E. NOTA

13. Simplify: $\sqrt{20-\sqrt{20-\sqrt{20-\sqrt{20-\ldots}}}}$.

A. -5 B. -4 C. $2\sqrt{5}$ D. 5 E. NOTA

14. David's annual MAθ poem (a limerick!):

If the nth term of infinite Series A is always less than
the nth term of infinite Series B,
And the sum of Series A will certainly approach
+ infinity,
Even the strictest teacher will condone,
That this test has easily shown,
That Series B will absolutely diverge, yippee!

What convergence test is used in the preceding poem?

A. Ratio B. Integral
C. Root D. Comparison E. NOTA

15. If $\sin\theta+(\sin\theta)^2+(\sin\theta)^3+\ldots=\frac{3}{2}$, then $\sin\theta=?$

A. -3 B. $-\frac{3}{5}$ C. $\frac{3}{5}$ D. 3 E. NOTA

16. $\sum_{i=1}^{10}\left(\int_{i}^{(i+1)}2x\,dx\right)=?$

A. 1 B. 99 C. 120 D. 121 E. NOTA

17. Determine the coefficient of the 4th term in the expansion of $(1+x)^{\frac{1}{2}}$.

A. $\frac{1}{16}$ B. $\frac{1}{6}$ C. $\frac{3}{8}$ D. 4 E. NOTA

18. The nth term of a sequence is the nth derivative of $f(x) = \sin x \cos x$ (for n=1,2,3...). Determine the sequence's 6th term.

A. -16cos(x) B. 8cos(2x)
C. -32sin(2x) D. 64sin(2x) E. NOTA

19. Let h(x,y) give the area of an ellipse with major axis length 2x and minor axis length 2y. Determine the value of $\sum_{i=0}^{\infty} h\left(4, \left(\frac{1}{2}\right)^i\right)$.

A. 4π B. 8π C. $\frac{16}{3}\pi$ D. diverges E. NOTA

20. A TV series is given a 20-week run in its first year. Each year the series exceeds, only meets, or falls under ratings expectations, the network will add 3 weeks, add 1 week, or subtract 2 weeks, respectively, from the series' run the following year. Assuming the show exceeds expectations 40% of the time and only meets them 20% of the time, what is the expected run of the show (in weeks) in its 11th year (assume no cancellation)?

A. 26 B. 26.6 C. 42 D. 50 E. NOTA

21. Use Newton's method twice to approximate a root of $x^3 - 4 = 0$. Start with $x_0 = 2$ and give the answer (x_2) to the nearest hundredth of a unit.

A. 1.59 B. 1.67 C. 2.87 D. 5.62

22. Evaluate the following limit: $\lim_{x\to 0} \frac{\frac{x^2}{2!} - \frac{x^4}{4!} + \frac{x^6}{6!} - \ldots}{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots}$.

A. 0 B. $\frac{1}{2}$ C. 1 D. diverges E. NOTA

23. A sequence is formed as numbers from successive rows of Pascal's triangle. (i.e. the first 10 terms are 1, 1, 1, 1, 2, 1, 1, 3, 3, 1). What is this sequence's 100th term?

A. 14 B. 1,001 C. 1,287 D. 2,002 E. NOTA

24. For each n let $\beta_n = \{x_0, x_1, \ldots, x_n\}$, be a partition of $[a,b]$. Suppose that as n approaches ∞ the largest subinterval in β_n approaches 0. For each k between 1 and n let t_k be some number on $[x_{k-1}, x_k]$. Determine $\lim_{n\to\infty}\left(\sum_{k=1}^{n}(t_k)^2 \Delta x_k\right)$.

A. $2b - 2a$ B. $b^2 - a^2$
C. $a^3 - b^3$ D. $\frac{1}{3}(b^3 - a^3)$ E. NOTA

25. Let f(x)=cosx. The last term in the Taylor series used to approximate $f\left(\frac{2\pi}{7}\right)$ with an error less than 0.00001 is $\frac{f^{(n)}(0)}{(n)!}\left(\frac{2\pi}{7}\right)^n$. Find the value of n.

A. 8 B. 9 C. 10 D. 11 E. NOTA

26. I will soon start work at three different jobs (A, B and C). I am developing the following work/rest pattern. In the first cycle, I will work a day each at job A, B, C (consecutively), with a day of rest after each day of work. In subsequent cycles, I will add a day at each job before having a day of rest (so the days of the first two cycles are spent: job A, rest, B, rest, C, rest, A, A, rest, B, B, rest, C, C, rest). What will I do on the 100th day of this pattern?

A. Job A B. Job B C. Job C D. Rest E. NOTA

27. Determine the convergence of $\sum_{n=1}^{\infty}(-1)^{n+3}\left(\frac{1}{5n-2}\right)$.

A. diverges B. converges radially
C. converges absolutely D. converges conditionally
E. NOTA

28. Determine the limit of the following sequence as n approaches infinity:
$1, 2, 2\frac{1}{3}, \ldots, 3 - \frac{2}{n}, \ldots$?

A. 1 B. 3 C. 5 D. diverges E. NOTA

29. Determine $\sum_{n=2}^{\infty}\frac{1}{(n-1)(n+2)}$, if it exists.

A. $\frac{1}{18}$ B. $\frac{1}{2}$ C. $\frac{11}{18}$ D. $\frac{11}{6}$ E. NOTA

30. Determine the interval of convergence of $\sum_{n=1}^{\infty}\frac{\sqrt[3]{n}}{2^n}x^n$.

A. (-2, 2) B. [-2, 2) C. (-1, 1) D. [-1, 1) E. NOTA