

Alpha Complex Numbers
FAMAT State Competition 2000

The abbreviation **NOTA** denotes "None Of These Answers."
For all problems, $i = \sqrt{-1}$.

1. $i^{4004} =$
- A. 1 B. -1
C. i D. $-i$ E. NOTA

2. The conjugate of $(4 - i)$ is which of the following?
- A. $\frac{1}{4-i}$ B. $-4 + i$
C. $4 + i$ D. $-4 - i$ E. NOTA

3. If $(2i + 6)(4i - 1) = ai + b$ then give the value of b .
- A. -14 B. -2
C. 2 D. 14 E. NOTA

4. Simplify: $(1 - i)^{10}$.
- A. 32 B. 1024
C. $-32i$ D. $1024i$ E. NOTA

5. For $f(x) = \sqrt{4x - 1}$ which of the following values are not real?
- A. $f(0.25)$ B. $f(f(4))$
C. $f(f(1))$ D. $f(f(0.25))$
E. NOTA

6. Which of the following is NOT a complex number?
- A. i^5 B. $\sqrt{3}i + 5$
C. $\sqrt{3}i$ D. $\sqrt{3}$ E. NOTA

7. Let $f(x) = a + bi$ when a is the number of letters in a given word/name x and $b = (2 + a)$. For which word/name is one-half times the value of f equal to $3 + 4i$?
- A. Christian B. Drew
C. Joshy D. Suresh
E. None of these answers

8. Which of the following is equivalent to $(2 + 2\sqrt{3}i)$?
- A. $2cis30^\circ$ B. $4cis60^\circ$
C. $4cis120^\circ$ D. $2\sqrt{2}cis30^\circ$
E. NOTA

9. The graph of $f(x) = (x - 2)(x + 4)$ has two real roots. Let $g(x) = f(x) + k$ for some real number k . For which value of k is it true that $g(x)$ has two complex roots of the form $a + bi$, for $b \neq 0$?
- A. 0 B. -6
C. 7 D. 10 E. NOTA

10. For $f(x) = 3x^4 + 6x^3 - 2x - 3$ give the value of $f(i)$.
- A. $-8i$ B. $4i$
C. $-6 + 8i$ D. $6 - 4i$ E. NOTA

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11. For $f(x) = \sec x + i \tan x$ and $g(x) = \csc x - i \cot x$, find the value of $f\left(\frac{\pi}{3}\right) \cdot g\left(\frac{\pi}{6}\right)$.

- A. 1 B. $4 - 3i$
 C. 7 D. $4 + 3i$ E. NOTA

12. Which is a complex root of $y = x^3 - 1$?

- A. $\frac{1+i\sqrt{3}}{2}$ B. $\frac{1-i\sqrt{3}}{2}$
 C. $\frac{-1+i\sqrt{3}}{2}$ D. $\sqrt{3}i$
 E. NOTA

13. Which of the following is NOT a fourth root of i ?

- A. $\text{cis } 22.5^\circ$ B. $\text{cis } 47.5^\circ$
 C. $\text{cis } 112.5^\circ$ D. $\text{cis } 292.5^\circ$
 E. NOTA

14. If

$$f(a, b) = \begin{cases} a + bi & \text{if } a \in \text{Reals and } b \in \text{Reals} \\ a + b & \text{if } a \notin \text{Reals or } b \notin \text{Reals} \end{cases}$$

which of the following is equivalent to $f(2, 3) + f(2, i)$?

- A. $5 + 2i$ B. $7 + i$
 C. $4 + 3i$ D. $4 + 4i$
 E. NOTA

15. Which of the following is equivalent to $(4 \text{cis } 120^\circ)(2 \text{cis } 30^\circ)$?

- A. $8 \text{cis } 3600^\circ$ B. $8 \text{cis } 150^\circ$
 C. $6 \text{cis } 150^\circ$ D. $16 \text{cis } 3600^\circ$
 E. NOTA

16. $\prod_{k=1}^{10} \frac{ki}{k+1} =$

- A. $\frac{1}{10}$ B. $\frac{-1}{10}$
 C. $\frac{1}{11}$ D. $\frac{-1}{11}$ E. NOTA

17. If $f(x) = 2x^2 + kx + 3$, then for what value of k does f have two complex, non-real roots?

- A. $|k| < 2\sqrt{6}$ B. $k < 2\sqrt{6}$
 C. $k < 24$ D. $|k| < 24$
 E. NOTA

18. For which equation could the graph of $f(x)$ have NO real roots? ($a \in \text{Reals}$).

- A. $f(x) = x^5 + x^3 - ax + 1$
 B. $f(x) = -x^4 + x^3 + ax + 1$
 C. $f(x) = -x^3 + x^2 + ax + 1$
 D. $f(x) = x^2 + ax + 1$
 E. NOTA

19. Let $f(x) = (x + i)^3$. When $x=2$, $f(x) = x^3 + C$. Which expression is equal to C ?

- A. $3 - 2i$ B. $15 + 12i$
 C. $-6 + 11i$ D. $1 + 12i$
 E. NOTA

20. Let $f(a, b) = a + bi$. Which is an expression for $f(2, 3 + 2i)$?

- A. $3i$ B. $5 + 2i$
 C. $4 + 3i$ D. $1 + 2i$ E. NOTA

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21. For what value of k is it true that

$$\sqrt{2 + ki\sqrt{2}} = 0 ?$$

- A. $-\sqrt{2}$ B. $i\sqrt{2}$
C. $\sqrt{2}$ D. $-i\sqrt{2}$ E. NOTA

22. $2e^{\left(\frac{\pi}{3}i\right)} =$

- A. $1 + i\sqrt{3}$ B. $2 + 2i\sqrt{3}$
C. $1 - i\sqrt{3}$ D. $2 - 2i\sqrt{3}$
E. NOTA

23. The graph of $f(x+1)$ has x -intercepts -1 and 3 . If f is a quadratic function, then which is a root of $g(x) = f(x) + 5$?

- A. $2 + i$ B. $3 + i$
C. $4 + i$ D. $5 + i$ E. NOTA

24. If $f(x) = \frac{x}{1+x}$ and $f(a) = \frac{7+i}{10}$ then which is the value of a ?

- A. $i - 7$ B. $i + 2$
C. $i + 3$ D. $i + 10$ E. NOTA

25. For some positive integers a , b and c , $((i^a)^b)^c = -1$. When the product abc is written in base 4, what is the units digit of the base 4 representation?

- A. 0 B. 1
C. 2 D. 3 E. NOTA

26. Let $(\sqrt{2x})(\sqrt{8x}) = a$. If x is a real number less than zero, then which must be true of a ?

- A. a is real and $a > 0$
B. a is real and $a < 0$
C. a is imaginary (of the form bi , $b \neq 0$)
D. a could be either real or imaginary
E. NOTA

27. $\sum_{n=1}^6 (-1)^n (i)^n =$

- A. 0 B. $1 - i$
C. 1 D. $1 + i$ E. NOTA

28. For the complex number $z = a + bi$, Let $f(z)$ be defined as the absolute value of the complex number z . Which equation is not true?

- A. $f(4 - 3i) = f(4 + 3i)$
B. $f(2i) = f(2)$
C. $f(4 + 3i) = f(3 + 4i)$
D. $f(i\sqrt{5}) = 5$
E. NOTA

29. Solve for x over the Complex Numbers:

$$x^{\frac{4}{3}} + 8x^{\frac{2}{3}} - 9 = 0$$

- A. $\{\pm 3i, \pm 1\}$ B. $\{\pm 27, \pm i\}$
C. $\{\pm 27i, \pm 1\}$ D. $\{\pm i, \pm 3i\}$ E. NOTA

30. If $2(\cos \theta + i \sin \theta) = \sqrt[3]{x}$ and $3(\cos \alpha + i \sin \alpha) = \sqrt[3]{y}$ then which is an expression for $x + y$?

- A. $8cis(3\theta) + 27cis(3\alpha)$
B. $8cis^3\theta + 27cis^3\alpha$
C. $6cis(3\theta) + 9cis(3\alpha)$
D. $6cis^3\theta + 9cis^3\alpha$
E. NOTA