Alpha Complex Numbers

FAMAT State Competition 2000

Solutions:

1. $i^{4004} = i^0 = 1$. Choice **A**.
2. **C**
3. $8i^2 - 2i + 24i - 6 = -14 + 22i$. Choice **A**
4. $(1-i)^{10} = ((1-i)^2)^5 = (-2i)^5 = -32i^5$ $= -32i$. Choice **C**.
5. f(0.25)=0; f(f(4))=f($\sqrt{15}$) and $4\sqrt{15} - 1 > 0$. Likewise, f(f(1)) is real. But f(f(0.25))=f(0) which is imaginary. Choice **D**
6. All are complex, of the form a+bi. In choice D, b=0 but $\sqrt{3}$ is complex. The answer is **E**.
7. $\frac{1}{2}f(x) = 3 + 4i$ so $f(x) = 6 + 8i$ and the word must have 6 letters. Choice **D**.
8. $4(cis 60) = 4(\cos 60 + i \sin 60) = 4(\frac{1}{2} + \frac{\sqrt{3}}{2}i)$ which simplifies to choice **B**.
9. $f(x) = x^2 + 2x - 8$ and the minimum value is at x=-1 and the value is -9. If the graph is raised more than 9 units then there will be no x-intercept and therefore no real roots. Choice **D** is the answer.
10. $f(i) = 3i^4 + 6i^3 - 2i - 3 = 3 - 6i - 2i - 3$ which simplifies to choice **A**.
11. $\sec\frac{\pi}{3} + i\tan\frac{\pi}{3} = 2 + i\sqrt{3}$. $\csc\frac{\pi}{6} - i\cot\frac{\pi}{6} = 2 - i\sqrt{3}$. The product is therefore $4 - 3i^2 = 7$, choice **C**.
12. Use cis and DeMoivre's theorem or divide $x^3 - 1$ by $(x-1)$ and get $x^2 + x + 1$. Use the quadratic formula to get roots $\frac{-1 \pm i\sqrt{3}}{2}$ so the answer is choice **C**.
13. $(i)^{\frac{1}{4}} = (cis\, 90)^{\frac{1}{4}} = cis\frac{90}{4} = cis\, 22.5$. The other roots are $cis\, 112.5, cis\, 202.5, cis\, 292.5$. Choice **B**.
14. $f(2,3) = 2 + 3i$, $f(2,i) = 2 + i$. The sum is 4+4i which is choice **D**.
15. **B**
16. $\frac{i}{2} \bullet \frac{2i}{3} \bullet \frac{3i}{4} \bullet \ldots \bullet \frac{10i}{11}$ which simplifies to $\frac{1}{11}i^{10} = \frac{1}{11}i^2 = \frac{-1}{11}$ which is choice **D**.
17. $k^2 - 4(6) < 0$ (using the discriminant) gives $k^2 < 24$, $|k| < 2\sqrt{6}$ which is **A**.
18. All odd degree functions pass through the x-axis and have a real root. Choice B has f(0)=1 and there is no min to the function. Choice **D** is the answer since a could be 0.
19. $x^3 + 3x^2 i + 3xi^2 + i^3 =$ $12i + 6i^2 + -i = 11i - 6$ which is choice **C**.
20. $2 + (3 + 2i)i = 2 + 3i - 2 = 3i$ which is choice **A**.
21. $ki\sqrt{2} = -2$ and $k = \frac{-2}{i\sqrt{2}}$ which rationalizes to $\sqrt{2}\,i$ which is choice **B**.
22. $e^{i\theta} = cis\theta$ so the expression is $2cis\frac{\pi}{3}$ or $2(\frac{1}{2} + i\frac{\sqrt{3}}{2})$ which simplifies to choice **A**.
23. $f(x+1) = (x+1)(x-3)$ so $f(x) = (x-1+1)(x-1-3) = x(x-4)$ and $f(x) + 5 = x^2 - 4x + 5$ which has roots $\frac{4 \pm 2i}{2} = 2 \pm i$. A root is choice **A**.
24. $\frac{x}{x+1} = \frac{7+i}{10}$, which gives $10x = 7 + 7x + i + ix$. Group x terms and factor and get $x(10 - 7 - i) = 7 + i$ so $x = \frac{7+i}{3-i}$. Rationalizing, gives $2 + i$ which is choice **B**.
25. For i to any power to be equal to -1, we have to have the power, when divided by 4, to have a remainder of 2. So the units place, in base four, is 2. Choice **C**.
26. $a = \sqrt{2x}\sqrt{8x} = i\sqrt{2|x|} \bullet i\sqrt{8|x|} = i^2\sqrt{16x^2}$ $= (-1)(4|x|)$ so a is less than zero and real. Choice **B**.

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Solutions : page two of two.

27. $-i + i^2 - i^3 + i^4 - i^5 + i^6 =$
    $-i - 1 + i + 1 - i - 1 = -i - 1$. Choice E.
28. Choice D is the answer:
    $\sqrt{\sqrt{5}^2 + 0^2} = \sqrt{5}$

29. $(x^{\frac{2}{3}} + 9)(x^{\frac{2}{3}} - 1) = 0$ so
    $x^{\frac{2}{3}} = -9$ *or* $x^{\frac{2}{3}} = 1$ and
    $x = (-9)^{\frac{3}{2}}$ *or* $x = 1^{\frac{3}{2}}$ so
    $x = \pm 27i^3 = \pm 27i$ and $x = \pm 1$
    which is choice *C*.
30. $2cis\theta = \sqrt[3]{x}$ so $x = (2cis\theta)^3$
    and by DeMoivre's Th. we get
    $8cis(3\theta) = x$ and likewise
    $y = 27cis(3\alpha)$. So (x+y) = choice **A**.