

Analytic Geometry  
FAMAT State Convention 2000

For all questions, answer E "NOTA" means none of the above answers are correct.

- Line AB contains the points  $(-3, 7)$  and  $(1, -9)$ . Line CD contains the point  $(4, 5)$  and is perpendicular to line AB. Find the value of the y-intercept of line CD.  
A) -1      B) 2      C) 4      D) 6      E) NOTA
- Find the sum of the coordinates of the focus of the graph of  $y + 1 = x^2 - 6x + 10$ .  
A)  $\frac{11}{4}$       B)  $\frac{13}{4}$       C)  $\frac{15}{4}$       D)  $\frac{17}{4}$       E) NOTA
- Find a unit vector in the same direction as the vector  $\langle 4, 2, -4 \rangle$ .  
A)  $\langle \frac{2}{5}, \frac{1}{5}, -\frac{2}{5} \rangle$       B)  $\langle \frac{2}{3}, \frac{1}{3}, -\frac{2}{3} \rangle$       C)  $\langle 2, 1, -2 \rangle$       D)  $\langle 40, 20, -40 \rangle$       E) NOTA
- The graph of the polar equation  $r^2 = 2 \sec(2\theta)$  can be categorized as a  
A) lemniscate      B) spiral      C) hyperbola      D) circle      E) NOTA
- An equation of the circle which contains the points  $(8, 3)$ ,  $(4, -5)$ , and  $(-4, -1)$  is  
A)  $x^2 + y^2 - 16x + 12y = 19$   
B)  $x^2 + y^2 + 4x - 6y = 27$   
C)  $x^2 + y^2 - 12x + 2y = 3$   
D)  $x^2 + y^2 - 4x - 2y = 35$   
E) NOTA
- Find the area of the graph determined by the parametric equations  $x = 4 \cos(t)$  and  $y = 3 \sin(t)$ .  
A)  $12\pi$       B)  $25\pi$       C)  $120\pi$       D)  $144\pi$       E) NOTA
- Rounded to the nearest tenth of a degree, find the value of  $\theta$ , where  $\theta$  is the acute angle by which the conic section  $8x^2 + 2y^2 + 3xy + 2x = 1$  must be rotated in order to eliminate the  $xy$  term.  
A)  $10.9^\circ$       B)  $13.3^\circ$       C)  $26.5^\circ$       D)  $31.7^\circ$       E) NOTA
- Find an equation of the locus of points which are equidistant from the points  $(5, 2)$  and  $(1, 4)$ .  
A)  $y = -\frac{1}{2}x + \frac{9}{2}$   
B)  $y = \frac{1}{2}x + \frac{3}{2}$   
C)  $y = -2x + 9$   
D)  $y = 2x - 3$   
E) NOTA

9. Rounded to the nearest tenth, find the shortest distance between the point  $(-2, -1)$  and the circle with equation  $x^2 - 8x + y^2 - 12y + 44 = 0$ .
- A) 1.2      B) 3.6      C) 6.4      D) 9.2      E) NOTA
10. Find the sine of the acute angle between the vector  $3\vec{i} - 4\vec{j}$  and vector  $24\vec{i} - 7\vec{j}$ .
- A)  $\frac{22}{75}$       B)  $\frac{3}{5}$       C)  $\frac{4}{5}$       D)  $\frac{24}{25}$       E) NOTA
11. Consider the graph of  $f(x) = 6 \cos(2x) + 12 \sin(2x)$ . Which of the following statements are true?
- i) The period of the graph is  $\pi$ .  
 ii) The amplitude of the graph is 18.  
 iii) The point  $(\frac{\pi}{2}, -6)$  is on the graph.  
 iv)  $f(x)$  is an even function.
- A) i, iii only      B) ii, iii only      C) i, ii, iii only      D) i, ii, iii, iv      E) NOTA
12. The graph of the equation  $r = \frac{11}{3 + 5 \cos(\theta)}$  can be classified most specifically as a
- A) cardioid      B) ellipse      C) limaçon      D) hyperbola      E) NOTA
13. To the nearest tenth of a degree, find the acute angle formed by the intersection of the graphs of the lines with equations  $y = 9x - 6$  and  $x = 2y - 7$ .
- A)  $20.2^\circ$       B)  $22.4^\circ$       C)  $57.1^\circ$       D)  $67.6^\circ$       E) NOTA
14. Find the product of the coordinates of the centroid of triangle JKL, given  $J(2, 4)$ ,  $K(7, -3)$ , and  $L(9, 5)$ .
- A) 10      B) 12      C) 24      D) 54      E) NOTA
15. Find the coordinates of the center of the circle with equation  $x^2 - 6x + 8y = 15 - y^2$ .
- A) (3, 4)      B) (-3, 4)      C) (3, -4)      D) (-3, -4)      E) NOTA
16. Given the equation  $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$ , where  $a, b, h,$  and  $k$  are rational numbers and  $b > a > 1, h > 1, k > 1$ , which of the following are necessarily true?
- i) The graph is an ellipse.  
 ii) The major axis lies along the x-axis.  
 iii) The coordinates of the center of the graph is at  $(h, k)$ .  
 iv) The eccentricity of the graph is greater than one.
- A) i, ii only      B) i, iii only      C) i, iii, iv only      D) i, ii, iii, and iv      E) NOTA

17. Find the shortest distance between the focus of the parabola with equation  $y = -\frac{1}{4}x^2 + \frac{1}{2}x - \frac{5}{4}$  and the line with equation  $4y = 3x - 1$ .
- A) 1      B)  $\sqrt{2}$       C)  $\sqrt{3}$       D) 2      E) NOTA
18. Given the rational function with equation  $f(x) = \frac{x^2 - 7x + 10}{x^3 - 6x^2 + 5x}$ , how many of the following statements are true?
- i) There is a "hole" in the graph at the point  $(5, \frac{3}{20})$ .
- ii) The graph has one horizontal asymptote.
- iii)  $f(x)$  has two real zeros.
- iv) The graph has three vertical asymptotes.
- v) The graph passes through all four quadrants.
- A) 1      B) 2      C) 3      D) 4      E) NOTA
19. In triangle  $\triangle PQR$ , angle  $\angle P$  is bisected by  $\overline{PS}$ . Given that  $PQ = 8$ ,  $SR = 2$  and  $QS = PR$ , find the perimeter of  $\triangle PQR$ .
- A) 16      B) 18      C) 20      D) 24      E) NOTA
20. Find the volume enclosed by the tetrahedron with vertices at the points  $(2, 0, -1)$ ,  $(12, 8, -4)$ ,  $(-8, 2, 9)$  and the origin.
- A) 12      B) 24      C) 36      D) 72      E) NOTA
21. Find the sum of the slopes of the asymptotes to the graph of the curve given by the equation  $16x^2 + 64x - 25y^2 + 400y - 1136 = 0$ .
- A)  $\frac{4}{5}$       B)  $\frac{16}{25}$       C)  $\frac{5}{4}$       D)  $\frac{25}{16}$       E) NOTA
22. Paul is building a parabolic arch. Building inspector Denise asks Paul how high the arch is at its center. Paul explains that his ladder is only 200 feet tall and will not reach the top, so he does not know. Denise takes the ladder 4 feet to the right of the center and climbs vertically upward to 192 feet and reaches the top of the arch 4 feet from its center. She then climbs down the ladder and walks 4 more feet to the right (a total of 8 feet from the center) and reaches the right edge of the arch (at 0 feet ground level). Denise amazes Paul by telling him the exact height of the center of the arch. What is this height that Denise told Paul?
- A) 216      B) 225      C) 248      D) 256      E) NOTA
23. Find the area of a parallelogram with a pair of adjacent sides  $\vec{a}$  and  $\vec{b}$ , where  $\vec{a} = 3\vec{i} + 2\vec{j} - 2\vec{k}$  and  $\vec{b} = -6\vec{i} + 8\vec{j} + 6\vec{k}$ .
- A) 23      B) 37      C) 46      D) 55      E) NOTA

24. Dennis, Carole, and Zubo are playing a math game to test the great Zubo's ability. First, Dennis yells out a point in cylindrical coordinates and then Carole yells out a point in spherical coordinates. Zubo magically transforms each of the coordinates to rectangular and finds the distance in 3-space between the two points he was given, then says his answer to the two amazed friends. Assuming Zubo is never wrong, what will Zubo give as his answer if, in a particular round, Dennis gives the point  $(3, \frac{2\pi}{3}, -5)$  and Carole gives the point  $(6, \frac{\pi}{6}, \frac{\pi}{3})$ ?
- A)  $\sqrt{34}$     B) 6    C)  $\sqrt{89}$     D) 10    E) NOTA
25. Find the area of the rectangle inscribed in the ellipse  $16x^2 + 25y^2 = 400$ , if the sides are the two latera recta and the two parallel line segments formed by connecting the endpoints of the latera recta.
- A)  $\frac{168}{5}$     B)  $\frac{182}{5}$     C)  $\frac{192}{5}$     D)  $\frac{208}{5}$     E) NOTA
26. Find the slope of the tangent line to the curve  $y = 4 \sin(\theta)$  at the point where  $\theta = \frac{3\pi}{2}$ .
- A) -2    B)  $-\sqrt{3}$     C)  $\sqrt{3}$     D) 2    E) NOTA
27. Let A equal the radius of the circle circumscribed about a triangle with side lengths 6, 7, and 11. Let B equal the radius of the circle inscribed in a triangle with side lengths 6, 7, and 11. Find  $\frac{B}{A}$  as a fraction in lowest terms.
- A)  $\frac{20}{77}$     B)  $\frac{29}{79}$     C)  $\frac{31}{83}$     D)  $\frac{44}{89}$     E) NOTA
28. Given point A = (1, 3, 2), point B = (4, 2, 5), point C = (3, 3, 1), and point D = (3, 1, -1), find the volume of the parallelepiped with edges  $\overline{AB}$ ,  $\overline{AC}$ , and  $\overline{AD}$ .
- A) 7    B) 14    C) 22    D) 26    E) NOTA
29. How many of the following statements are true about the graph given by the equation  $r = 2 \cos(2\theta)$ ?
- i) The graph is a rose with two petals.  
 ii) The graph is completely traced one time on  $[0, \pi)$ .  
 iii) The graph is symmetric about the polar axis.
- A) 1    B) 2    C) 3    D) 0    E) NOTA
30. When graphed on the Argand plane, which of the following complex numbers does not lie along a circle of radius 5 centered at the location of the complex number  $4 - 3i$ ?
- A)  $1 + i$     B)  $3 - 7i$     C) 0    D)  $8 - 6i$     E) NOTA