1. Which of the following is (are) always true when $0 \le x < 2\pi$.

I. $\sin^2(x) + \cos^2(x) = 1$

II. $1 + \tan^2(x) = \sec^2(x)$

III. $1 + \cot^2(x) = \csc^2(x)$

IV. $\sin(x) = \dfrac{\sin^2(x)}{\sin(x)}$

A) I only B) I, II, and III only C) I and IV only D) All true E) NOTA

2. Find the sum of the roots of $f(x)$, if $f(x) = x^3 + 3x^2 - 5x + 1$.

A) -3 B) -1 C) 1 D) 3 E) NOTA

3. What does the Arctan(tan($3\pi/4$)) equal?

A) $\dfrac{-3\pi}{4}$ B) $\dfrac{-\pi}{4}$ C) $\dfrac{\pi}{4}$ D) $\dfrac{3\pi}{4}$ E) NOTA

4. What is the shortest distance between the lines $y - 2x = 9$ and $y - 2x = -7$?

A) 1 B) $\sqrt{5}$ C) $\dfrac{16}{5}$ D) $\dfrac{16\sqrt{5}}{5}$ E) NOTA

5. Find $a + b + c + d + e + f$, if $\sin(7.5^O)$ is expressed exactly and simplified as $\dfrac{\sqrt{a - b\sqrt{c} - d\sqrt{e}}}{f}$.

A) 16 B) 24 C) 28 D) 36 E) NOTA

6. Find $a^4 + n^2$ if $\left(\dfrac{\sqrt{1-\cos(2x)}}{\sin(x)}\right) \cdot \left(\dfrac{1-\cos(2x)}{2}\right)$ is expressed in the form $a \cdot \sin^n(x)$.

A) 3 B) 6 C) 7 D) 16 E) NOTA

7. Expand $\cos(5x)$ in the form $a \cdot \cos^b(x) + c \cdot \cos^d(x) + e \cdot \cos^f(x)$, where a, b, c, d, e, f are integers then find a + b + c + d + e + f .

A) 10 B) 12 C) 18 D) 20 E) NOTA

8. Find the area of a triangle with sides of lengths 6, 20, and 24.

A) $\frac{5\sqrt{95}}{2}$ B) $5\sqrt{95}$ C) $\frac{20\sqrt{4290}}{2}$ D) $20\sqrt{4290}$ E) NOTA

9. Find the hundredths digit of the natural log of the angle (in radians) between the vectors $\langle 1,-3,4,2 \rangle$ and $\langle -3,2,4,-1 \rangle$.

A) 1 B) 2 C) 3 D) 4 E) NOTA

10. The polar graph, r, is a(n)?

$$r = \frac{12}{6 + 4\cos(\Theta)}$$

A) Cycliod B) Ellipse C) Hyperbola D) Parabola E) NOTA

11. Suppose Laura is on an 8 x 8 tile grid, where the bottom left hand tile is the tile (0,0).. The tile (x,y) is to be interpreted as the tile x units to the right of the tile (0,0) and y units above the tile (0,0). Laura starts at the tile (0,0), and her goal is to reach the tile (8,8) by only moving one unit to the right or moving upward one unit, at a time. If there is a trapdoor on the tile (6,4) that would cause Laura to not achieve her goal, what is the probability that Laura will not reach her goal?

A) $\frac{1}{10}$ B) $\frac{7}{429}$ C) $\frac{211}{12870}$ D) $\frac{4}{23}$ E) NOTA

12. Simplify: $\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)^{(-6i)}$

A) $e^{-4\pi}$ B) $e^{-\pi}$ C) e^{π} D) $e^{4\pi}$ E) NOTA

13. $M = \begin{bmatrix} -1 & 2 & 6 & 9 \\ 2 & 1 & -4 & 3 \\ 1 & -3 & 4 & 1 \\ 1 & -1 & 7 & 7 \end{bmatrix}$ What does $\det([M]^{-1} \cdot [M]^{T})$ equal?

A) -4 B) -2 C) 0 D) 1 E) NOTA

14. Find $\sum_{x=0}^{n} x$ if $\sum_{x=0}^{n} x^3 = 44100$.

A) 190 B) 210 C) 231 D) 253 E) NOTA

15. Find the closed form of the summation below, where $n \geq 2$.

$$\sum_{x=1}^{\infty} \frac{x}{n^x}$$

A) $\frac{n}{(n-1)^2}$ B) $\frac{n}{n+1}$ C) $\left(\frac{1}{n}\right)^{(n-1)}$ D) $\frac{n-1}{n^2}$ E) NOTA

16. Find the 45th Fibonacci number where the two leading terms in the sequence are one.

A) 1,046,689,139 B) 1,123,488,453 C) 701,408,733
D) 1,134,903,170 E) NOTA

17. If $(x+y)^7$ is expanded and like terms are NOT combined, what is the probability that a randomly selected term will contain an x term with an exponent less than 3?

A) $\frac{15}{16}$ B) $\frac{99}{128}$ C) $\frac{1}{2}$ D) $\frac{29}{128}$ E) NOTA

18. There is a hat containing 5 red marbles, 3 blue marbles, and 2 yellow marbles. Suppose Ovid reaches his hand into the hat and grabs 3 marbles one at a time. If it is given that the first marble is a yellow marble, then what is the probability that the remaining two marbles grabbed are the same color?

A) $\frac{13}{36}$ B) $\frac{13}{45}$ C) $\frac{15}{36}$ D) $\frac{1}{2}$ E) NOTA

19. An airplane is at a fixed altitude and is heading 60 degrees west of north at a speed of 500 mph. As the plane reaches an arbitrary point it encounters a wind with a velocity of 80 mph in the direction 75 degrees north of east. What is the speed of the plane (in mph) while the plane is subject to this windy condition? (Round to the nearest thousandth.)

A) 526.408 B) 526.409 C) 574.578 D) 574.579 E) NOTA

20. Ugwechi found that a person's physical condition when expressed by a number, zero (death) to one hundred, exponentially increases from age zero (birth) to age twenty-five. After the age of twenty-five until death, the physical condition of a person decreases linearly. Given the data in the chart below, what was the subject's physical condition at the age of 24?

Age	*Physical Condition*
0	*1*
30	*20*
100	*0*

Note: The subject died at age 100.

A) 20 B) 21 C) 22 D) 23 E) NOTA

21. What is the remainder when 7^{401} is divided by 5?

A) 1 B) 2 C) 7 D) 9 E) NOTA

22. If $\sin(x)$ intersects the x - axis three times on the bounded region $[0,2\pi]$, then how may times does $\sin(5x)$ intersect the x - axis on the bounded region $[0,2\pi]$?

A) 10 B) 11 C) 12 D) 13 E) NOTA

23. What is the amplitude of the function $f(x)$, if $f(x) = 3 \cdot \sin(x) + 4 \cdot \cos(x)$?

A) 1 B) $\sqrt{7}$ C) 5 D) 7 E) NOTA

24. If **u**, **v**, and **w** are vectors, which of the following is (are) always true?
    I. $\mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}$
    II. $\mathbf{u} \times \mathbf{v}$ always exists
    III. If $\| \mathbf{u} \times \mathbf{v} \| = 1$ then the two vectors, **u** and **v**, have the same direction.
    IV. $|\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})| = |\mathbf{v} \cdot (\mathbf{u} \times \mathbf{w})|$ if **u**, **v**, and **w** are all three dimensional non-zero vectors.

A) I only B) I and III C) IV only D) II and IV E) NOTA

25. What is the surface area of the 3 - D figure ABCD, if A = (1,2,3) B = (4,−1,3) C = (4,-4,4) D = (8,1,1), rounded to the nearest square unit?

A) 15 B) 16 C) 17 D) 18 E) NOTA

26. Find the area enclosed by the following polar graph, r.

$$r = \frac{36}{5\cos^2(\Theta) + 4}$$

A) 6π B) 24π C) 30π D) 36π E) NOTA

27. How many contiguous zeros are at the end of the expansion of 2003!, when expressed in base 17?

A) 101 B) 123 C) 141 D) 499 E) NOTA

28. Evaluate: $\lim_{x\to 0} \frac{\sin(2x)}{x}$

A) Does not exist B) 0 C) 2 D) 4 E) NOTA

29. If $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e^x$, then what does $\lim_{x\to\infty}\left(1+\frac{3}{4x}\right)^{2x}$ equal?

A) $e^{\frac{3}{4}}$ B) $e^{\frac{3}{2}}$ C) e^3 D) $e^{\frac{9}{2}}$ E) NOTA

30. Solve for x where $x \in \Re$

$$3^x = 9^{\frac{x^2+2x+1}{2}}$$

A) $x = \{0\}$ B) $x = \left\{\frac{-1\pm\sqrt{3}}{2}\right\}$ C) $x = \left\{\frac{-1\pm i\sqrt{3}}{2}\right\}$ D) $x = \{\}$ E) NOTA