

Some questions have been borrowed from former competition tests.

No calculator allowed.

Choice E. NOTA means none of the given answers are correct.

1. If $f(x) = 4x^2 + 3x + 6$ and $g(x) = x - 3$, find $(f \circ g)(x)$.
- A) $4x^2 - 21x + 33$ B) $4x^2 - 3x + 4$
 C) $4x^2 - 3x + 3$ D) $4x^2 - 31x + 42$
 E) NOTA
2. What is the period of the function $f(x) = \cos^4 x - \sin^4 x$?
- A) $\frac{\pi}{4}$ B) $\frac{\pi}{2}$
 C) π D) 2π E) NOTA
3. A farmer has 160 ft of fencing to enclose 2 adjacent rectangular pigpens that are the same size and have one common side. What dimensions should be used for each pig pen so that the enclosed area will be a maximum?
- A) $4\sqrt{15}$ ft. by $\frac{8}{5}\sqrt{15}$ ft.
 B) 40 ft. by $\frac{80}{3}$ ft.
 C) 20 ft. by $\frac{80}{3}$ ft.
 D) 40 ft. by 40 ft.
 E) NOTA
4. If A is a 2×2 matrix and the determinant of A is 10, then the determinant of A^{-1} is
- A) 10 B) 0.10 C) 100
 D) 0.01 E) NOTA
5. For $\cos(\alpha + \beta) = \frac{1}{2}\cos\beta - \frac{\sqrt{3}}{2}\sin\beta$, which of the following could be the value of α ?
- A) $\frac{\pi}{3}$ B) $\frac{\pi}{6}$
 C) $\frac{-\pi}{3}$ D) $\frac{-\pi}{6}$ E) NOTA
6. If $e^{\frac{\pi i}{3}} = a + bi$ and a and b are real, then $a = \underline{\hspace{1cm}}$.
- A) 0 B) $\frac{1}{2}$
 C) $\frac{\sqrt{3}}{2}$ D) e E) NOTA
7. Assuming convergence, find the value of the fraction $\frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \dots}}}$.
- A) $1 + \sqrt{2}$ B) $1 - \sqrt{2}$ C) $-1 - \sqrt{2}$
 D) $-1 + \sqrt{2}$ E) NOTA
8. Which function(s) is/are even?
- i) $y = \sin\left(\frac{1}{x}\right) + 1$
 ii) $y = \cos(|x|) + 1$
 iii) $y = |\tan(x + 1)|$
- A) i only B) i, ii only
 C) i, ii, iii D) ii only E) NOTA

9. Find $a + b + c$ given the following.

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

- A) $\frac{-4}{3}$ B) $\frac{4}{3}$
 C) $\frac{3}{4}$ D) $\frac{-3}{4}$ E) NOTA

10. What is the complete solution set for x if

$$\sqrt{\frac{(\tan^2 x + 1)^2}{\sec^4 x}} = 1 \quad ?$$

- A) For all real values of x
 B) All reals, x , such that $x \neq \frac{\pi}{2}k$; $k \in \text{Integers}$
 C) All reals, x , such that $x \neq \frac{\pi}{2}k$; $k \in \text{Odd Integers}$
 D) All reals, x , such that $x \neq \frac{\pi}{2}k$; $k \in \text{Positive odd integers}$
 E) NOTA

11. Evaluate the sum $\sum_{n=0}^{33} \cos\left(\frac{n\pi}{8}\right)$.

- A) $\frac{\sqrt{2+\sqrt{2}}}{2}$ B) $\frac{\sqrt{2+\sqrt{3}}}{2}$
 C) $1 + \frac{\sqrt{2+\sqrt{2}}}{2}$ D) $\frac{-\sqrt{2+\sqrt{2}}}{2}$ E) NOTA

12. What is half of 2^8 ?

- A) 1^8 B) 2^4 C) 2^6 D) 2^7 E) NOTA

13. Evaluate: $\sin\left(\text{Arctan}\frac{-3}{4} + \text{Arccos}\frac{5}{13}\right)$

Note: domain and range of the inverse functions are assumed to be the standard domain and range for the Arc-functions.

- A) $\frac{16}{65}$ B) $\frac{33}{65}$
 C) $\frac{56}{65}$ D) $\frac{63}{65}$ E) NOTA

14. Find the y-intercept of the slant asymptote for the following equation:

$$y = \frac{2x^3 + 8x^2 - 22x - 60}{x^2 - 5x + 6}$$

- A) 4 B) 9
 C) 18 D) 24 E) NOTA

15. What is the period of: $y = -3 \tan 2x + 1$

- A) $\frac{-\pi}{3}$ B) 2π C) 3π D) $\frac{\pi}{2}$ E) NOTA

16. The solution set of the inequality

$x^2 - 4x - 5 \geq 0$ is either (1) $A \leq x \leq B$ or (2) $x \geq C$ or $x \leq D$. Choose the correct solution set (1 or 2 above) and then choose the corresponding answer:

- A. $A = -1, B = 5$ B. $A = -5, B = 1$
 C. $C = 5, D = -1$ D. $C = 1, D = -5$
 E. NOTA

17. If $\frac{1}{\sin x + \cos x} = 3$ then give the value of $\sin(2x)$.

- A. $-\frac{8}{9}$ B. $-\frac{2}{9}$
 C. $\frac{1}{9}$ D. $\frac{1}{3}$ E. NOTA

18. For $f(x) = |x-1|$ and $g(x) = |x+1|$ the graph of $y = f(g(x))$ is reflected over the line $x=0$. If the reflection is the function $h(x)$, which is equal to $h(2)$?

- A. 2 B. 1
 C. 0 D. -2 E. NOTA

19. A parabola has focus at the point $(2, 4)$ and directrix with equation $x = -6$. Which is the x-intercept of the graph of the parabola ?

- A. -3 B. -1
 C. 1 D. $4 + 4\sqrt{2}$
 E. NOTA

20. If $(199)(201)$ is expressed as $(0.01x-1)(0.01x+1)$ then $\frac{3x}{10} =$

- A. 6000 B. 2000
 C. 0.6 D. 0.2 E. NOTA

21. If x is an integer and $8 < x < 20$, then how many different triangles are there with lengths of 8 and 19 and x ?

- A. 11 B. 10 C. 9
 D. 8 E. NOTA

22. $\log_2(x) = \log y$ for $x > 1, y > 1$. Which is an expression for $\log_y x$?

- A. $-\log 2$ B. 2
 C. $1 - \log 5$ D. $1 - \log 2$
 E. NOTA

23. For a fixed angle Θ such that $0 < \Theta < \frac{\pi}{2}$

and $\tan \Theta = \frac{3}{4}$, what can be said about the zeros of the graph of $4x^2 + 4x \sin \Theta - \cos^2 \Theta = y$?

- A. There is one real, double zero.
 B. There are two distinct irrational zeros.
 C. There are two non-real zeros.
 D. There are two distinct rational zeros.
 E. NOTA

24. If a triangle with sides 8, 12 and 5 has an area of $\frac{A\sqrt{B}}{C}$ in reduced radical form, then $A + B + C =$

- A. 26 B. 25
 C. 34 D. 38 E. NOTA

25. For $0 < x < \frac{\pi}{2}$

$$2 \tan x + 4 \tan^2 x + 8 \tan^3 x + \dots = 10.$$

Give the positive value of $\sin(x)$.

- A. $\frac{5}{11}$ B. $\frac{\sqrt{5}}{11}$ C. $\frac{\sqrt{146}}{11}$
 D. $\frac{5\sqrt{146}}{146}$ E. NOTA

26. 40 grams of a 10% saline solution (10% salt, 90% pure water) are mixed with a 40% saline solution, to form a 20% saline solution. How many grams of the 40% solution are used?

- A. 30 B. 20
 C. 15 D. 10 E. NOTA

27. $S = \left\{ \frac{1}{3}, \sqrt{3}, 1 \right\}$

Let **K** be the least possible value of x for which 3^{1-x} is a member of the set S .

Let **J** be the greatest possible value of x for which 9^x is a member of the set S .

Give the value of **J+K**.

- A. 0 B. $\frac{3}{4}$
 C. $\frac{7}{8}$ D. 1
 E. NOTA

28. The number $100!$ is divided by 6^n and has a remainder of zero. What is the greatest possible integer value of n ?

- A. 52 B. 50
 C. 48 D. 2 E. NOTA

29. If $\sqrt{9^{5x+3}} = \sqrt[3]{9}$ then what is the value of $\sqrt[7]{2^{30x}}$?

- A. $-\sqrt{2}$ B. $\frac{-\sqrt{2}}{2}$
 C. 2 D. $\frac{1}{4}$ E. NOTA

30. If $|2x+6| + |x-1| = 10$ then give the sum of all possible values of x which are solutions over the real numbers.

- A. $\frac{-10}{3}$ B. $\frac{5}{3}$
 C. $\frac{65}{3}$ D. $\frac{80}{3}$ E. NOTA