FLORIDA INVITATIONAL ALGEBRA2 TEAM SOLUTIONS February 23, 2008

1. A: $\frac{-A}{B}=-\frac{1}{6}$ B: $\left(\frac{14}{3},0\right)$ C: $\left(0,\frac{7}{9}\right)$ D: $6x-y=29.$

B: Let y=0 and solve for x. $\left(\frac{14}{3},0\right)$. Question asked for ordered pair.

C: Let $x=0$ and solve for y. $\left(0,\frac{7}{9}\right)$. Question asked for ordered pair.

D: Since the slope of the line is $-\frac{1}{6}$, the slope of the perpendicular will be 6. $6x-y=29.$

2. A: $-1+14i$ B: $-\frac{1}{5}-\frac{7i}{5}$ C: $5+2i$ D: $\sqrt{17}$

A: $(2+3i)^2+(3-i)(1+i)=(4+12i+9i^2)+(3+2i-i^2)=(-5+12i)+(4+2i)=-1+14i$.

B: Multiplying by the conjugate of the denominator gives $\frac{1-3i}{2+i}\bullet\frac{2-i}{2-i}=-\frac{1}{5}-\frac{7i}{5}$.

C: $2i^{31}-5i^{82}=5-2i$. The conjugate of this is $5+2i$. All answers were to be $a+bi$ form.

D: $\left|(3+6i)-(4+2i)\right|=\left|-1+4i\right|=\sqrt{17}$

3. A: $\left(-\frac{5}{4},-\frac{49}{8}\right)$ B: $(-3,0),\left(\frac{1}{2},0\right)$ C: $\left(-\frac{5}{4},-6\right)$ D: $y=-\frac{25}{4}$

A: $y=2\left(x^2+\frac{5}{2}x+\frac{25}{16}\right)-3-\frac{50}{16}; y=2\left(x+\frac{5}{4}\right)^2-\frac{49}{8}$ giving a vertex of $\left(-\frac{5}{4},-\frac{49}{8}\right)$

B: To find the x-intercepts, let y=0. Factor as $(2x-1)(x+3)$ to find the solutions as $(-3,0),\left(\frac{1}{2},0\right)$.

Question asked for ordered pairs.

C: To find the focus we must first find the value of p. $a=2,$ so $2=\frac{1}{4p}, p=\frac{1}{8}$. The parabola

Opens up so we want to add p to the y-coordinate of the vertex making the focus $\left(-\frac{5}{4},-6\right)$.

D: The parabola opens up, so the directrix would be a horizontal line. Subtract the value of p from The y-coordinate of the vertex to give the equation $y=-\frac{25}{4}$.

4. A: 4 B: -110 C: $t=\frac{2L-6}{P}$ D: 63 or 63_8

A: Squaring both sides of $x=\sqrt{12+\sqrt{12+\ldots}}$ gives $x^2=12+\sqrt{12+\sqrt{12+\ldots}}$ however $\sqrt{12+\sqrt{12+\ldots}}=x$. Substituting gives $x^2=12+x$. Solving gives $x=4,-3$. The value of x must be positive so reject the -3.

B: $\left(\frac{(12)2-4}{2}\right)\left(\frac{(-8)10-30}{10}\right)=(10)(-11)=-110$

C: $L=\frac{Pt}{2}+3;$ multiply through by 2 gives $2L=Pt+6$.Solving for t gives $\frac{2L-6}{P}$.

4 continued D: $43_{12} = 4(12)+3 = 51_{10}$. Changing to base 8, the highest power of 8 in 51 is 8. There are 6 eights with 3 left over. So our answer is 63_8 .

5. A: $\frac{128}{5}$ B: $\left(-1,\frac{1}{4}\right)$ C: $(-3,-1)$ D: $(-\infty,-4] \cup [\frac{5}{3},\infty)$

A: Clearing parentheses and multiplying by 6 to eliminate fractions gives the equation

$9x+102 = 30+14x-56; x = \frac{128}{5}$

B: $-4 < 2x+8 < 16$ gives the solution $(-6,4)$

$4 < -8x+6 < 14$ gives the solution $\left(-1,\frac{1}{4}\right)$ .Since this is a conjunction, we want the intersection of the two sets which is $\left(-1,\frac{1}{4}\right)$.

C: $x \neq 0$ $-1 < \frac{2x+3}{x} < 1;$ Solving $-1 < \frac{2x+3}{x}; -x < 2x+3; x > -1.$

Solving $\frac{2x+3}{x} < 1; 2x+3 < x; x < -3$. Put these critical points on the number line and testing the zones shows that the numbers between -1 and -3 satisfy the inequality.

D: $3x^2+7x-12 \geq 8$; $3x^2-7x-20 \geq 0;$ factoring gives $(3x-5)(x+4)$ which makes the critical Points $\frac{5}{3}, -4;$ Plot these on a number line and test the zones. Solutions is $(-\infty,-4] \cup [\frac{5}{3},0)$

6. A: 9 B: 84 C: 15 D: 5

A: $x = 24, y = 12, z = 16; \frac{24 \bullet 12}{2 \bullet 16} = 9$

B: $\frac{x}{y} = \frac{3}{4}; x = \frac{3}{4}y$; $\frac{x-6}{y+2} = \frac{3}{5}; 5x-30 = 3y+6;\ 5 \bullet \frac{3}{4}y - 30 = 3y+6; y = 48, x = 36; x+y = 84$

C: $\frac{x}{y} = \frac{3}{7}; x = \frac{3}{7}y; x+y < 60; \frac{3}{7}y + y < 60; x < 18$. Since x must be a multiple of 3, the smallest value for x is 15 since x and y must be integers.

D:For $5x+12 = 60$, when $x = 0, y = 5.$ These values would give the smallest value of $\sqrt{x^2+y^2}$ which would be 5.

7. A: 8 B: $2q+p$ C: a^2 D: -2

A: $\log_{12}(x-5)(x+4) = 1; 12 = x^2-9x+12; x^2-9x+8 = 0, x = 8, \not{1}$

B: $\log 175 = \log 25 + \log 7 = 2\log 5 + \log 7 = 2q+p$

C: $z = \log_b a + \frac{1}{\log_a b}; z = \frac{\log_b a \log_a b + 1}{\log_a b}; z = \frac{2}{\log_a b}; z\log_a b = 2; \log_a b^z = 2; a^2 = b^z$

D: $\left(\frac{\log 125}{\log 25}\right) - \left(\frac{\log 5}{\log \sqrt{5}}\right) + \left(\frac{\log 5\sqrt{5}}{\log \frac{1}{5}}\right)$ Changing to base 5 gives $\frac{3}{2} - \frac{1}{\frac{1}{2}} + \frac{\frac{3}{2}}{-1} = -2$

FLORIDA INVITATIONAL ALGEBRA2 TEAM SOLUTIONS February 23, 2008

8. A: 13 B: 8 C: 7 D: 77

A: $F(0)=2, F(1)=3, F(n+2)=2F(n)-F(n+1)$. Let $n=0; F(2)=F(0)-F(1)=1;$
$F(2)=1$. Let $n=1, F(3)=2F(1)-F(2)=5.\ F(3)=5.$
Let $n=2, F(4)=2F(2)-F(3)=-3. F(4)=-3.$ Let $n=3, F(5)=2F(3)-F(4)=13.$

B: Let $u=\frac{1}{x}, v=\frac{1}{y}$. Substituting gives $\frac{u}{4}+\frac{7v}{2}=\frac{5}{4}$ and $\frac{u}{2}-3v=-\frac{5}{14}$. Clearing denominators gives the system $u+14v=5$ and $7u-42v=-5$. Solving this gives $u=1, v=\frac{2}{7}$. And $x=1, y=\frac{7}{2}$. The value of $x+2y=1+2\bullet\frac{7}{2}=8$.

C: Substitute the values for x and y from the point $(2,-1)$ into each equation and solve for the values of k and j. $2k-7=-1, k=3; 4+j=8, j=4. k+j=7.$

D: Using synthetic division

$$\begin{array}{r|rrrrr} -2 & 2 & -5 & 0 & -1 & 3 \\ & & -4 & 18 & -36 & 74 \\ \hline & 2 & -9 & 18 & -37 & 77 \end{array}$$

9. A: $y=\pm\frac{2}{3}x$ B: 4 C: 4π D: 20

A: Moving 36 to the other side and dividing by -36 gives $\frac{y^2}{4}-\frac{x^2}{9}=1$, a hyperbola with center (0,0). The slopes would be $\pm\frac{2}{3}$ making the equations of the asymptotes $y=\pm\frac{2}{3}x.$

B: Solve the system $x^2+y^2=16, x^2=3y-12.$ Substituting for x^2 gives $3y-12+y^2=16.$
Moving everything to the same side and factoring gives the solutions $-7, 4$. Reject the $-7.$
The parabola and the circle intersect at $(0,4)$ so the value of y is 4.

C: Putting this equation in standard form gives $\frac{y^2}{2}+\frac{x^2}{8}=1.\ a=\sqrt{2}, b=2\sqrt{2}, ab\pi=4\pi.$

D: $\frac{x^2}{64}-\frac{y^2}{36}=1, c=10.$ Therefore, the distance from foci to foci is 20.

10. A: $\frac{1}{81}$ B: $-22680x^4y^3$ C: -10 D: 2

A: Solving the equation gives $x=-\frac{4}{3}; 27^{-\frac{4}{3}}=\frac{1}{81}$

B: The 4th term would be found by $\frac{7!}{4!3!}\bullet(3x)^4(-2y)^3=-22680x^4y^3$

C: $3xA-2A+Bx+4B=-28;$ giving the system $3A+B=0, -2A+4B=-28;$ solving
This system gives $A=2, B=-6; A+2B=-10$.

D: Since we know that -2 is a solution, use synthetic division to get the quadratic
$x^2-2x+2=0.$
Solving this gives the solutions as $1\pm i$. The difference is $2i$. The absolute value of $2i$ is 2.

FLORIDA INVITATIONAL ALGEBRA2 TEAM SOLUTIONS February 23, 2008

11. A: $\frac{11}{6}$ B: $6a+3h-2$ C: -7 D: 13

A: $R=6, S=-11, T=-35.$ This gives the quadratic $6x^2-11x-35=0.$ Factoring gives

$(3x+5)(2x-7)=0; x=-\frac{5}{3},\frac{7}{2}.$

B:

$$\frac{f(a+h)-f(a)}{h}=\frac{3(a+h)^2-2(a+h)+1-(3a^2-2a+1)}{h}=\frac{3a^2+6ah+3h^2-2a-2h+1-3a^2+2a-1}{h}=$$

$6a+3h-2$.

C: $2x^2-7x-15=-22; 2x^2-7x+7=0; b^2-4ac=49-4\bullet 2\bullet 7=-7$.

D: Evaluating by expansion by minors using the first column gives

$$1\begin{vmatrix}-1 & 4\\ -3 & 8\end{vmatrix}-2\begin{vmatrix}2 & -1\\ -3 & 8\end{vmatrix}+5\begin{vmatrix}2 & -1\\ -1 & 4\end{vmatrix}=4-26+35=13.$$

12. A: 2 B: $\frac{a+b}{ab}$ C: x^2+3x+4 D: 4

A: $3^{2x+y}=3^2, 2^{x+2y}=2^4$; Since the bases are equal, set the exponents equal and solve the system $2x+y=2, x+2y=4. x=0, y=2.$ Sum is 2.

B: $$\left(\frac{1}{a^{-1}+b^{-1}}\right)^{-1}=\left(\frac{1}{\frac{1}{a}+\frac{1}{b}}\right)^{-1}=\left(\frac{1}{\frac{b+a}{ab}}\right)^{-1}=\frac{a+b}{ab}$$

C: $f(x-1+1)=f(x)$ so to find $f(x)$ substitute $x+1$ for x.

$f(x+1)=(x+1)^2+x+1+2=x^2+3x+4$.

D: $\sqrt{y+5}=7-y;$ squaring both sides gives $y+5=9-14y+y^2$. Solving gives $y=11,4.$ Reject the 11 as it does not satisfy the original equation.

13. A: 1 B: -2 C:0 D:2

First, since the y-intercept is 0, that is the value of c. Now we want to write a system using the values of the x-intercepts and the value of c. Using the x-intercept of -2 and substituting the -2 for x gives the equation $8=4a-2b+0$ or $8=4a-2b.$ Doing the same for the x-intercept of 1 gives $-1=a+b+0$. Solving this system gives $a=1, b=-2.$ We now have the values for a, b, & c. To find $P(-1)$ we must write the polynomial and substitute -1 for x.

$P(x)=x^3+x^2-2x; P(-1)=-1+1+2=2$.

14. A: $\sqrt{2}$ B: 1 C: $y=\frac{5}{3}x+\frac{11}{3}$ D: $4x^2-5$

A: $\frac{1}{y-1}=y+1;$ multiplying through by $y-1$ gives $1=y^2-1$. Solving gives $\sqrt{2}$.

B: Multiplying through by 9 gives the quadratic $x^2-6x+9=0.$ The solution is 3 which Makes the value of $\frac{x}{3}=1$.

C: The slope of the original equation is $-\frac{3}{5}$ so the slope of the perpendicular line is $\frac{5}{3}$ giving the Equation $y = \frac{5}{3}x + b$. Substitute the point $(-1, 2)$ in for x and y to find the y-intercept is $\frac{11}{3}$.

D: $f(g(x)) = 4(x^2 - 2) + 3 = 4x^2 - 5.$