

1.  $A = 3(69) + 89 = 296$ ;  $B = 2\sqrt{8^2 + 15^2} = 34$ ;  $C = (4 \cdot 8)^2 + (3 \cdot 15)^2 = 3049$ ;  $D = \sqrt{(5 \cdot 8)^2 + (2 \cdot 15)^2} = 50$   
 $A + B + C + D = 3429$

2. Use a 2-sample t-test; we want  $\mu_{\text{Sophia}} > \mu_{\text{Liz}} \rightarrow t < \frac{-4}{\sqrt{\frac{4.9^2 + 6.6^2}{15}}} \rightarrow t < -1.88 \rightarrow p \approx 0.035$

3. A is the definition of the standard deviation of a probability distribution  $\rightarrow A$  to 2 places =  $C = 2.07$ ; B is just  $\left(\frac{5}{12}\right)(0.15 + 0.15 + 0.1 + 0.1) = \frac{5}{24}$ ;  $\left|C - \frac{1}{B}\right| = \frac{273}{100}$  or 2.73.

4. For part A: if it's a uniform probability density curve, then two sides must lie parallel to the x-axis and two sides parallel to the y-axis;  $(0, 0)$ ,  $(x, 0)$ ,  $(0, f(x))$ , and  $(x, f(x))$  are the vertices. Hence, the area will be  $xf(x)$ ; setting

$2x - x^3 = 1$  gives solutions  $x = 1, x = \frac{-1 + \sqrt{5}}{2}, x = \frac{-1 - \sqrt{5}}{2}$ . However,  $x = \frac{-1 - \sqrt{5}}{2}$  is extraneous.

For parts B&C: a continuous probability density curve implies that at  $x = 1$ ,  $x = 2k - kx \rightarrow k = 1$ ; the value of  $n$  which would make this a probability density curve is  $n = 2$ . Thus,  $1 + 1 + 2 + \frac{-1 + \sqrt{5}}{2} = \frac{7 + \sqrt{5}}{2}$ .

5. Since  $\sigma$  must be positive,  $x > 0; y > 0; \sigma^2_{M+N} = x^2 + y^2 < 25$ ; quarter-circle of radius 5  $\rightarrow$  area =  $25\pi/4$ .

6.  $A = \binom{10}{7}(0.3)^7(0.7)^3$ ;  $B = 1 - (0.7)^{10}$ ;  $C = \binom{10}{9}(0.3)^9(0.7)^1 + (0.3)^{10}$ ;  $D = \text{binomcdf}(10, .3, 6) -$

$\text{binomcdf}(10, .3, 4) = \binom{10}{4}(0.3)^4(0.7)^6 + \binom{10}{4}(0.3)^5(0.7)^5 + \binom{10}{6}(0.3)^6(0.7)^4$ ;  $A + B + C + D = 1.321$

7. The expected value of a single roll is  $\left(\frac{1}{9}\right)(3)(1 + 3 + 5) + \left(\frac{2}{9}\right)\left(\frac{1}{2}\right)(2 + 4 + 6) = \frac{13}{3}$ . It can easily be seen that the trace of a  $k \times k$  matrix has  $k$  elements, so the expected value of the sum is  $\frac{13k}{3}$ .

8. The probability of Sida getting a full house with the remaining cards is  $\frac{12 \binom{4}{C_3}}{48 \binom{C_3}} (note that he has rigged the deck such that he cannot draw aces). There are 47 cards left for Lisa to get a full house; there are 11 possible cards for her to get a 3 of a kind from  $\rightarrow 11 \binom{4}{C_3}$  ways; there are now 11 cards left for her to get her pair from. Note that there is only 1 way to get 2 aces, compared to  ${}_4C_2 = 6$  for the rest of the cards, so  $10 \binom{4}{C_2} + 1 = 61$  ways to get her pair, with  ${}_{47}C_5$  possible hands; total probability is  $\frac{(12 \binom{4}{C_3})(11 \binom{4}{C_3})(61)}{(\binom{48}{C_3})(\binom{47}{C_5})} \approx 0.0000049$ .$

9. It's easiest to find A by just summing the first 50 triangular numbers, doubling that sum, then adding the required sum to account for the numbers which are both triangular and square (these are  $R_1, R_8,$  and  $R_{49}$ ). Thus,  $2 \sum_{n=1}^{50} \frac{n(n+1)}{2}$

$= \sum_{n=1}^{50} n^2 + n = \left(\frac{50(51)(101)}{6}\right) + \left(\frac{50(51)}{2}\right)$  (use the hints given in question 3). We need to add this to  $R_1 + R_8 + R_{49}$ , divide by 50, and round to the nearest integer; hence,  $A = 934$ .

The median of S is  $\frac{R_{25} + R_{26}}{2} = \frac{25(26) + 26(27)}{2} = 676$ . Now, use the hint from question 2 to find parts C and D:

note that  $\text{var}(X) = E(X^2) - (E(X))^2$ , and  $D = (E(X))^2$ ;  $C + D^2 = E(X^2) = \frac{1}{50} \sum_{n=1}^{50} n^2 \approx 859$ .

$A + B + C + D^2 = 2469$

10.  $a + b = 0.6$ ;  $a + 3b = 0.7 \rightarrow 2b = 0.1 \rightarrow b = 0.05 \rightarrow a = 0.55$ ;  $0.55 - 0.5 = 0.05$

11.  $c_{10} = \left(\sqrt{\frac{2}{9}}\right) \left(\frac{\Gamma(5)}{\Gamma(4.5)}\right) = 0.972804$ ;  $s \approx 2.78089 \rightarrow \frac{s}{c_{10}} = \sigma \approx 2.86$

12. A: There are 3 prime numbers and 3 nonprime numbers in [1, 6]  $\rightarrow$

$$3 \left( P(\text{prime}) \log_2 \left( \frac{1}{P(\text{prime})} \right) \right) + 3 \left( P(\text{nonprime}) \log_2 \left( \frac{1}{P(\text{nonprime})} \right) \right) = \frac{2}{3} \log_2 \left( \frac{9}{2} \right) + \frac{1}{3} \log_2 (9) \approx 2.50 = A$$

B:  $0.6 \log_2 \left( \frac{5}{3} \right) + 0.4 \log_2 \left( \frac{5}{2} \right) \approx 0.97$

C: First note that  $P(X = 5) = 0$  does not make this undefined; it simply means that  $X = 0$  isn't a possible outcome and should be ignored. It can easily be seen that  $k = 0.18$ , so  $0.18 \log_2 \left( \frac{1}{.18} \right) + 0.19 \log_2 \left( \frac{1}{.19} \right) + 0.2 \log_2 \left( \frac{1}{.2} \right)$

$$+ 0.21 \log_2 \left( \frac{1}{.21} \right) + 0.22 \log_2 \left( \frac{1}{.22} \right) \approx 2.32$$
;  $A + B + C = 5.79$ .

13. Note that  $55 = 2(3^3) + 1$ ,  $161 = 2(3^4) - 1$ ,  $488 = 2(3^5) + 2$ ,  $1460 = 2(3^6) + 2$ ; transformation (3) will be almost perfectly linear  $\rightarrow A = 3$

The LSRL of the raw data is  $y = 454.2x - 1502.9 \rightarrow B = 4542$ ,  $C = -15029$

The LSRL of the transformed data ( $y$  vs.  $3^x$ ) is  $y = 2.003t + .2857$ , where  $t = 3^x \rightarrow D + E = 10(2 + .3) = 23$

$$\frac{(4(4542) - 15029) \log_2 (3)}{23} \approx 216$$

14. Just plug them into the calculator. Note that the whole numbers are 0 through 9, not 1 through 10;  $\frac{AB}{CD} = 7.96$

15. For part A, use a 1-sided matched-pairs t-test  $\rightarrow A = 0.58$ ; for B use a 2-sided matched-pairs t-test  $\rightarrow B = 0.60$ ;  $A + B = 1.18$