January Regional: Calculus Individual Test
Let choice E) NOTA denote "None of the Above Answers is Correct"

1) Find the range of the function $f(x)=\dfrac{1}{1+\sin^2 x}$ over the domain of all real numbers.

A) $(0,\infty)$ B) [0, 1] C) $[0.5,\infty)$ D) [0.5, 1] E) NOTA

2) Evaluate: $\lim\limits_{n\to\infty}\dfrac{6^n+2^n}{8^n}$.

A) 0 B) $\dfrac{3}{4}$ C) 1 D) Diverges E) NOTA

3) A comedy lounge sells a maximum of 40 tickets per show. When the price of an individual ticket is \$20 the lounge sells all its tickets for the show. However, with each increase of \$5 in ticket price, the lounge sells one less ticket. What should the ticket price be in order to maximize profit?

A) \$90 B) \$100 C) \$110 D) \$120 E) NOTA

4) Evaluate the limit: $\lim\limits_{x\to-\infty}\left(\dfrac{-9+x}{x}\right)^{-x/3}$

A) 0 B) e^{-3} C) e^3 D) ∞ E) NOTA

5) The function $f(x)=e^{1/(x-1)}$ has one vertical asymptote at $x=a$ and one horizontal asymptote at $y=b$. Find $a+b$.

A) –1 B) 0 C) 1 D) 2 E) NOTA

6) With an initial guess of $x_1=0$, find the second iteration of Newton's Method when calculating a root to the equation $y=x^5-5x^2+3x+3$.

A) –1 B) $-\dfrac{2}{3}$ C) $\dfrac{1}{3}$ D) 2 E) NOTA

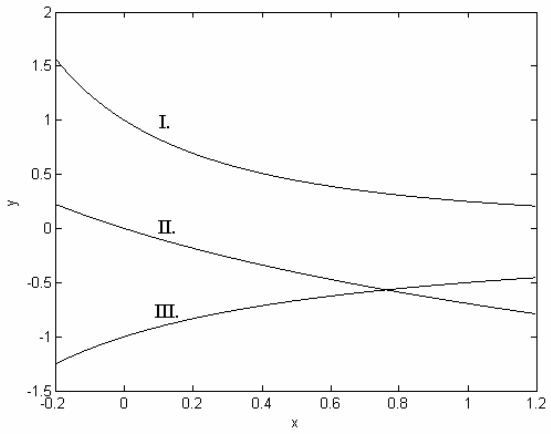
7) Find the maximum value of $\sin(\theta)+\cos\left(\theta+\dfrac{\pi}{6}\right)$ over the domain $0\le\theta\le\pi$.

A) $\dfrac{3}{4}$ B) $\dfrac{\sqrt{3}}{2}$ C) 1 D) $\dfrac{1+\sqrt{3}}{2}$ E) NOTA

8) Evaluate $\dfrac{d}{dx}\left[\dfrac{x^2+2x-1}{x^3+3x+1}\right]$ at $x=0$.

A) –5 B) –1 C) 0 D) $\dfrac{2}{3}$ E) NOTA

9) Shown are three functions I., II., and III. One of them is $f(x)$, one is $f'(x)$, and one is $f''(x)$. Which of the following correctly identifies each of the three graphs?



	$f(x)$	$f'(x)$	$f''(x)$
A)	I.	II.	III.
B)	II.	I.	III.
C)	II.	III.	I.
D)	III.	I.	II.
E)	NOTA		

10) The equation of the line tangent to $h(x)=x^3+2x$ when $x=1$ is $y=Ax+B$ where A and B are real numbers. Find A^2+B^2.

A) 9 B) 13 C) 18 D) 29 E) NOTA

11) Given: $\int_a^b g(x)dx=-2$, $\int_c^b g(x)dx=-3$, $\int_d^a g(x)dx=4$

Find: $\int_c^d g(x)dx$

A) –5 B) –3 C) –1 D) 9 E) NOTA

12) Which of the following Theorems/Definitions is used to show that $\lim_{x\to\infty}\frac{\cos(x)}{x}=0$?

A) Central Limit Theorem
B) $\delta-\varepsilon$ Limit Definition
C) Descarte's Theorem
D) Squeeze Theorem
E) NOTA

13) Find the first derivative of $g(x)=\frac{2x}{x^2+1}$.

A) $-\frac{2}{x^2}+2$ B) $\frac{2}{(x^2+1)^2}$ C) $\frac{-4x^2}{(x^2+1)^2}$ D) $\frac{-4x^2+2}{(x^2+1)^2}$

E) NOTA

14) Find the x-coordinate(s) of the point(s) of inflection of the function $y=\sqrt{x^3+2}$.

A) $x=-2$ only
B) $x=0$ only
C) $x=-2$ and $x=0$
D) The function does not have any points of inflection
E) NOTA

15) Given $\int_a^b \sin(x)dx = 1$. Find $\int_{a+2\pi}^{b+4\pi} \sin(x+3\pi)dx$.

A) –1 B) 0 C) 1 D) 2 E) NOTA

16) Use the linear approximation of $f(x) = \sqrt{x+1}$ near $x = 0$ to approximate $\sqrt{1.02}$.

A) 1.005 B) 1.01 C) 1.015 D) 1.02 E) NOTA

17) Given $h(x) = (x+1)^2 \cos^2(3x) e^{2x} (2x-1)^3$. Find $h'(0)$.

A) –2 B) 0 C) 2 D) 6 E) NOTA

18) Two lines normal to the graph $y = x^2$ are drawn. Denote line L_1 as the normal line at $x = 1$ and line L_2 as the normal line at $x = 2$. Find the *y*-coordinate of the point of intersection of lines L_1 and L_2.

A) –12 B) 2 C) $\frac{13}{2}$ D) $\frac{15}{2}$ E) NOTA

19) Given $f(x) = x^3 + x + 1$ and $g(x) = f^{-1}(x)$.

Evaluate: $\lim_{h \to 0} \frac{g(1+h) - g(1)}{h}$

A) $-\frac{4}{9}$ B) $\frac{1}{4}$ C) 1 D) 4 E) NOTA

20) A particle is moving along the *x*-axis whose position is defined by $s(t) = t^3 + 2t^2 - 4t - 8$. Over which of the following time intervals is the velocity of the particle both negative and decreasing?

A) $(-\infty, 2)$ B) $\left(-2, -\frac{2}{3}\right)$ C) $\left(-\frac{2}{3}, \frac{2}{3}\right)$ D) $\left(-2, \frac{2}{3}\right)$ E) NOTA

21) When the volume in cubic feet of an expanding sphere is increasing four times as fast as its radius in feet, the radius is how many feet long?

A) $\frac{1}{4\pi}$ B) $\frac{1}{2\sqrt{\pi}}$ C) $\frac{1}{\pi}$ D) $\frac{1}{\sqrt{\pi}}$ E) NOTA

22) How many of the following statements are TRUE?
I. A piecewise continuous function cannot be differentiable.
II. A function that is not differentiable cannot be continuous.
III. The sum of two or more functions that are not continuous cannot be continuous.
IV. The sum of two or more functions that are not differentiable cannot be differentiable.

A) 0 B) 1 C) 2 D) 3 E) NOTA

23) Use differentials to approximate $\sqrt{7}$ given that $\sqrt{9} = 3$.

A) $\frac{7}{3}$ B) $\frac{13}{5}$ C) $\frac{11}{4}$ D) $\frac{17}{6}$ E) NOTA

24) The height of a rocket launched into space is given by the function $h(t)=\begin{cases} at^3+bt^2+ct, & \text{if } 0\le t\le 1 \\ t^2+9t-4, & \text{if } t>1 \end{cases}$ where a, b, and c are real numbers.

If the height, velocity, and acceleration are all continuous functions, find $a^2+b^2+c^2$.

A) 18 B) 98 C) 194 D) 226 E) NOTA

25) Find the second derivative of $y=x^2e^x\ln(x)$.

A) $y''=2e^x$

B) $y''=(x^2+2x)e^x\ln(x)+xe^x$

C) $y''=(2x^2+6x+2)e^x\ln(x)+(3x+3)e^x$

D) $y''=(x^2+4x+2)e^x\ln(x)+(2x+3)e^x$

E) NOTA

26) Let $f(x)$, $g(x)$, and $h(x)$ be polynomials of degree n, m, and k, such that n, m, and k are all positive integers satisfying $n>m>k$. Define the polynomial $P(x)=f(x)[g(x)+h(x)]$. What will be the degree of the polynomial of the k^{th} derivative of $P(x)$, $\frac{d^k}{dx^k}[P(x)]$?

A) $n-k$ B) $n+m-k$ C) $n+m$ D) $nm-k$ E) NOTA

27) Let $y=\sin(x^2)$ and $x=\cos(t)+\sqrt{\pi}$. Find $\frac{dy}{dt}$ when $t=\frac{\pi}{2}$.

A) $-2\sqrt{\pi}$ B) 0 C) 1 D) $2\sqrt{\pi}$ E) NOTA

28) What is the average rate of change of the function $y=x^2+2x+6$ over the domain [0, 4]?

A) 2 B) 4 C) 6 D) 8 E) NOTA

29) Given $f(x)$ is an even function and $g(x)$ is an odd function. Which of the following are even functions?

I. $\frac{d}{dx}[f(x)g(x)]$ II. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]$ III. $\frac{d}{dx}[f(g(x))]$ IV. $\frac{d}{dx}[g(f(x))]$

A) I. and II. only B) I. and IV. only C) II. and III. only D) I. and IV. only

E) NOTA

30) Find the n^{th} derivative of $y=xe^x$.

A) $y^{(n)}=ne^x$

B) $y^{(n)}=(x+n)e^x$

C) $y^{(n)}=xne^x$

D) $y^{(n)}=x^ne^x$

E) NOTA