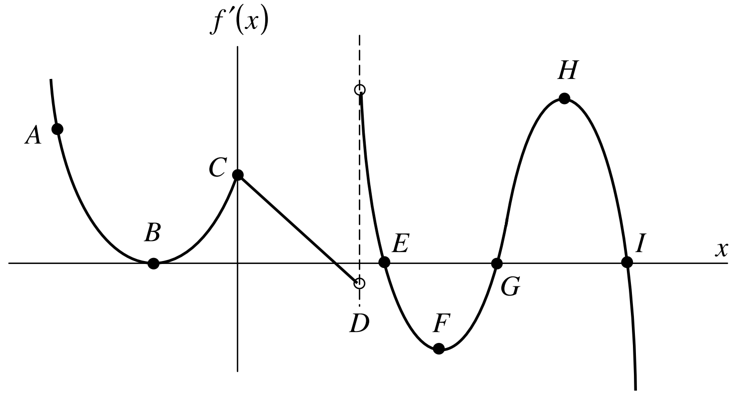
January Regional Calculus Team: Question #1

Shown on the graph below is the function $f'(x)$. The graph $f'(x)$ has horizontal tangents at points B, F, and H. It is also known that $f(x)$ is continuous. List the letters of the true statements in alphabetical order.



A. $f(x)$ is increasing at point A.

B. $f(x)$ is concave up at point B.

C. $f''(x)$ is differentiable at point C.

D. $f(x)$ is not differentiable at point D

E. $f(x)$ has a relative minimum at point E.

F. $f''(x)$ is zero at point F.

G. $f(x)$ is concave down at point G.

H. $f(x)$ has an inflection point at point H.

I. $f''(x)$ is increasing at point I.

January Regional Calculus Team: Question #2

Given: $3x^3 + y^2 = (x - y)^2$

Find the slope of the tangent line at $x = 1$.

January Regional — Calculus Team: Question #3

The table below gives values for $f(x)$, $g(x)$, and their derivatives $f'(x)$ and $g'(x)$.

x	1	2	3	4
$f(x)$	–3	1	4	2
$f'(x)$	–2	2	–6	–5
$g(x)$	4	3	1	–2
$g'(x)$	5	–1	–3	–1

Define the following functions:

$A(x)=g\left(\frac{1}{x}\right)$ $\quad B(x)=\frac{g(x)}{f(x)}$ $\quad C(x)=f(g(x))$ $\quad D(x)=g(f(x))$

Find: $A'(1)+B'(2)+C'(3)+D'(4)$

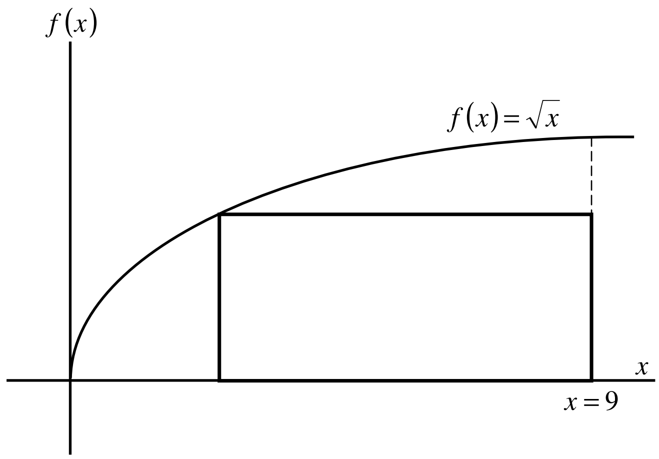
January Regional — Calculus Team: Question #4

The base of two different solids, S_1 and S_2, is the region enclosed by the line $y=x+1$, the x- and y-axes, and the line $x=1$. Let V_1 be the volume of solid S_1 that has square cross sections perpendicular to the x-axis and V_2 be the volume of solid S_2 that has square cross sections perpendicular to the y-axis.

Find V_1-V_2.

January Regional   Calculus Team: Question #5

A rectangle is inscribed in the region below the graph $f(x)=\sqrt{x}$ and above the *x*-axis. If the rightmost edge of the rectangle lies along the vertical line $x=9$ and the bottom edge lies along the *x*-axis, find the maximum area of the rectangle. Write your answer in simplest radical form.



January Regional   Calculus Team: Question #6

Given $g(x)=x^2-2x+1$ over the domain $1\le x\le 7$. Let $A=\int_1^7 g(x)dx$.

B = The approximation of A using the right endpoints of rectangles with 2 equal subdivisions.
C = The approximation of A using the left endpoints of rectangles with 6 equal subdivisions.
D = The approximation of A using the midpoints of rectangles with 3 equal subdivisions.

Find $A+B+C+D$.

January Regional **Calculus Team: Question #7**

Find the x-intercept of the line normal to the graph $h(x)=\frac{1}{3}x^2-\frac{1}{2}x+1$ that is also parallel to the line $y=-2x+9$.

January Regional **Calculus Team: Question #8**

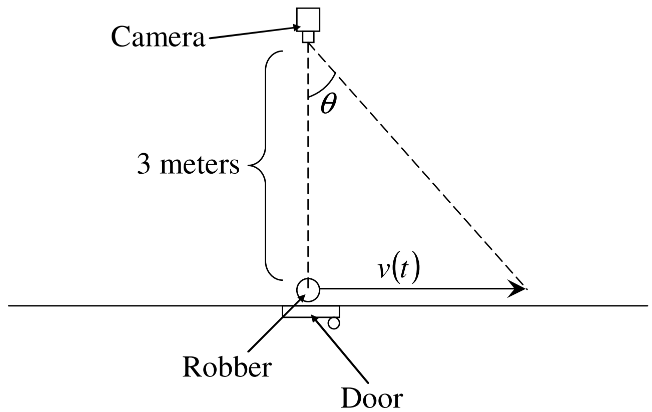
Let $s(t)=t^2+2t-3$, $1\le t\le 5$, be the distance traveled in meters by a runner running in a straight line. An observer tracks the runner's distance starting at 1 minute and ending at 5 minutes.

A = The average velocity of the runner measured in meters/sec between $t=1$ and $t=3$.
B = The instantaneous velocity of the runner measured in meters/sec at $t=2$.
C = The time in seconds at which the instantaneous velocity of the runner measured in meters/sec equals the average velocity measured in meters/second between $t=1$ and $t=5$.
D = The distance traveled by the runner measured in meters between $t=2$ and $t=4$.

Find $A+B+C+D$.

January Regional — Calculus Team: Question #9

A robber enters a bank by breaking into the front door. A camera is positioned such that it is initially facing the door directly 3 meters away. The robber travels with a velocity $v(t)=\frac{1}{2}t^2-\frac{1}{3}t$ where $v(t)$ is measured as the velocity traveling to the right in meters per second. The angle θ is shown.



Find the rate of change of the angle θ, $\frac{d\theta}{dt}$, in radians per second at $t=3$ seconds.

January Regional — Calculus Team: Question #10

List the letters of the true statements in alphabetical order.

A. $\frac{d}{dx}\left[\sqrt{x-1}\right]=\frac{1}{2\sqrt{x-1}}$ for all real values of x.

B. $f(x)=x^{1/3}-1$ is differentiable everywhere.

C. The maximum value of $g(x)=-x^2+4x+4$ is 8.

D. $\lim_{x\to\infty}\frac{2x-1}{\sqrt{5+2x^2}}=1$

E. A quadratic polynomial with positive real coefficients is never concave down.

F. $\lim_{x\to 2}\frac{x-2}{x^2-4}$ exists.

G. $h(x)=x^3-x$ is strictly increasing over the domain of all real numbers.

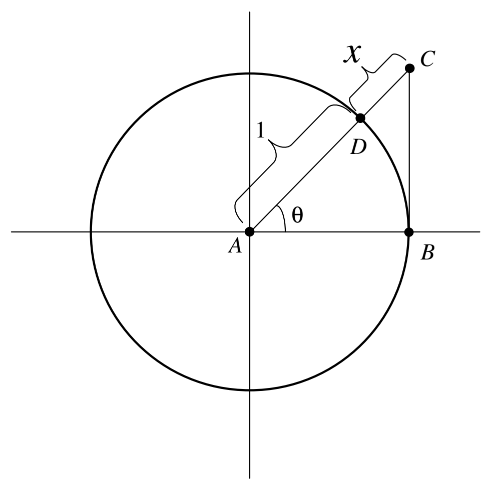
January Regional **Calculus Team: Question #11**

Given $f(x)=\sin(x)$ and $g(x)=\cos(x)$; $0<x<2008\pi$

A = The number of points of inflection of $f(x)$ over the domain $0<x<2008\pi$.
B = The number of relative minima of $f(x)$ over the domain $0<x<2008\pi$.
C = The number of relative maxima of $g(x)$ over the domain $0<x<2008\pi$.
D = The number of points where $f(x)=g(x)$ over the domain $0<x<2008\pi$.

Find: $(A+B)-(C+D)$

January Regional **Calculus Team: Question #12**



Shown is the unit circle and triangle ABC. The lengths of segments AB and AD are both equal to 1 and line segment BC is tangent to the unit circle at the point $(1,0)$.
Denote the length of line segment CD as x.

Evaluate: $\lim_{\theta\to 0}\frac{x}{\theta}$.

January Regional **Calculus Team: Question #13**

Find the slope of the line tangent to the graph $y = x^3 - 2x^2 + 4x - 3$ at its only point of inflection.

January Regional **Calculus Team: Question #14**

Find the sum of all values of c that satisfy the Mean Value Theorem for the function $h(x) = x^3 - 4x^2 - 4x + 2$ over the domain $0 \le x \le 2$.

Given $p(x)$ is a parabola. It is known that $\int_0^1 p(x)dx = \frac{1}{3}$ and that $p(x)$ has a vertex at the point $(2,-2)$.

Find $p(0)$