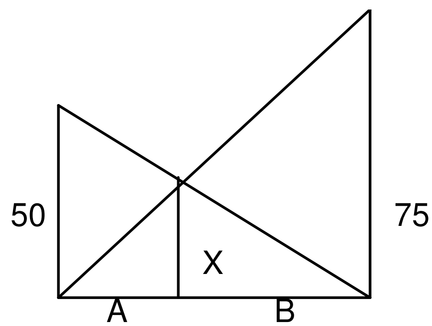
February Geometry Regional Solutions

1. **D**. It does not specify that the points must be non collinear.
2. **C**. The formula is n(n-3)/2, where n is the number of sides. A dodecagon has 12 sides, so the answer is 12(9)/2 = 54.
3. **D**. Since BC and AC are equal, angles A and B are congruent, so A also equals 6x+7. So 2(6x+7) + (4x+6) = 180. Solving the equation gives x = 10 and angle A is then 67.
4. **A**. Vertical angles are congruent. $(2x-3)^2 = 3x(x-2) \Rightarrow 4x^2 - 12x + 9 = 3x^2 - 6x$ $x^2 - 6x + 9 = 0 \Rightarrow x = 3$. When you plug 3 in to either angle, you get 9.
5. **C**. The area of a trapezoid is $\frac{h}{2}(b+b_1)$. The height is 15 and one of the bases is 32. To find the other base, use the 30-60-90 principle to find the length of the triangle base, which is $5\sqrt{3}$. So the length of the larger base of the trapezoid is $32+10\sqrt{3}$. Plugging the values into the formula produces the answer.
6. **D**. Use Heron's theorem to find the area of the triangle. Solving for the side lengths produces 4x+5x+6x=90, so x=6 and the length of each side is 24, 30 and 36. The semi perimeter is 45. Plugging into Heron's theorem produces $\sqrt{45(21)(15)(9)}$, which reduces to the solution.
7. **B**. The two angles add to 180 degrees. $x^2 - 7x + x^2 + 3x + 150 = 180$, which leads to $2x^2 - 4x - 30 = 0$. Solving by factoring gives x = 5 and x = -3. x = 5 produces a negative angle, but x = -3 produces 30 and 150 degrees. The complement of the smaller angle is 60 degrees.
8. **A**. If the interior angle is 135, the exterior angle is 45. The formula for the exterior angle of a polygon is 360/n, where n is the number of sides. So the number of sides must be 8, therefore the polygon is an octagon.
9. **C**. Since the area of the annulus is $825\pi$, the area of the larger circle is $841\pi$. The square root of 841 is 29, which is the radius of the larger circle. So the diameter is 58.
10. **B**. The sum of the two smallest sides of a triangle must be greater than the largest size. So, in this triangle, the third size must be $4 < x < 11$. Therefore, the answer is B.
11. **C**. It is a proportion problem. If every number is changed to inches, the equation is $\frac{\frac{7}{3}}{84} = \frac{x}{207}$. Solving for x gives you the solution.
12. **B**. Find the midpoint of YZ, which is M(2,9). The length XM, by the distance formula, is the solution.
13. **C**. The arithmetic mean is $\frac{x+y}{2} = 17 \Rightarrow x + y = 34$. The geometric mean is $\sqrt{xy} = 8 \Rightarrow xy = 64$. Substituting $y = \frac{64}{x}$ into the arithmetic mean gives $x + \frac{64}{x} = 34$ which leads to $x^2 + 64 = 34x$. Solving by factoring gives x = 2 and 32. The larger number is 32.
14. **A**.
15. **B**. The diagonals of a rhombus intersect and form right angles. The hypotenuse of each triangle is a side of the rhombus. The hypotenuse is 15, and 15x4 = 60.

16. **C**. The height of the building is opposite the 60 degree angle, so the distance Ted is from the building is opposite the 30 degree angle. Therefore, $x\sqrt{3} = 100 \Rightarrow x = \dfrac{100\sqrt{3}}{3}$

17. **A**. The inverse of the statement $p \Rightarrow q \;\; is \;\; \sim p \Rightarrow \sim q$. That is the solution.

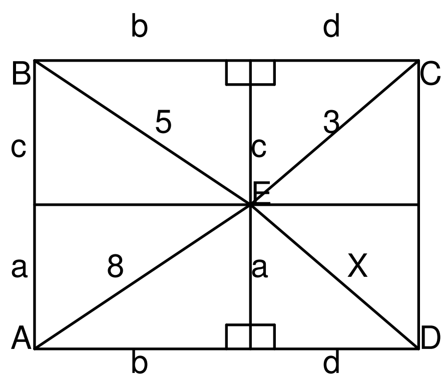
18. **D**. The formula is $|30H - 5.5M| = |30(11) - 5.5(8)| = 286$. Since the angle is acute, the answer is the solution.




19. **C**.

By using similar triangles, we can find the height between the wires, X. The first proportion is $\dfrac{50}{X} = \dfrac{A+B}{B}$. The second proportion is $\dfrac{75}{X} = \dfrac{A+B}{A}$. Solving each proportion gives 50B = X(A+B) and 75A = X(A+B). Therefore, 50B=75A or $B = \dfrac{3A}{2}$.

Substituting that into the second proportion gives $\dfrac{75}{X} = \dfrac{A + \frac{3A}{2}}{A} \Rightarrow \dfrac{75}{X} = \dfrac{\frac{5A}{2}}{A}$. This leads to $\dfrac{75}{X} = \dfrac{5}{2}$. Solving this proportion gives you the solution.




20. **B**.

Using the above diagram, you can create four right triangles. They are $b^2 + c^2 = 25,\; a^2 + b^2 = 64,\; c^2 + d^2 = 9,\; and\; a^2 + d^2 = X^2$. By solving the first two equations for $b^2$, $25 - c^2 = 64 - a^2$. Therefore, $a^2 - c^2 = 39$. Substituting the third equation in produces $a^2 - (9 - d^2) = 39 \Rightarrow a^2 + d^2 - 9 = 39 \Rightarrow a^2 + d^2 = 48$. Therefore, $X^2 = 48 \Rightarrow X = 4\sqrt{3}$.

21. **A**. The area of an equilateral triangle is $\frac{s^2\sqrt{3}}{4}$. Plugging in the values into the ratio produces $\frac{\frac{X^2\sqrt{3}}{4}}{\frac{\left(\left(3\sqrt{2}\right)X\right)^2\sqrt{3}}{4}} = \frac{\frac{X^2\sqrt{3}}{4}}{\frac{18X^2\sqrt{3}}{4}} = \frac{1}{18}$.

22. **D**. The slope of the line is the negative reciprocal of the original line, or $\frac{2}{5}$. The equation of the line in point slope form is $y-4=\frac{2}{5}(x+3)$. Solving for y gives the solution.

23. **D**. Find the dimensions of the larger rectangle formed by the garden and the sidewalk, which are 42 feet by 32 feet. The area of the garden and sidewalk is 1344. Subtract the area of the garden (600), and you have your solution.

24. **C**. Since there are 360 degrees in a circle, x+11x = 360. Therefore, x = 30. Since the area of the sector is $\frac{25\pi}{12}$, $\frac{25\pi}{12}=\frac{30}{360}\left(\pi r^2\right) \Rightarrow r=5$. The arc length of the sector is $\frac{30}{360}\left(2\pi(5)\right)=\frac{5\pi}{6}$. So the perimeter of the sector is 10 + $\frac{5\pi}{6}$.

25. **A**. The perimeter of the square is 14(4) = 56 by the 45-45-90 principle. The area of the square is 14(14) = 196. The ratio is $\frac{56}{196}=\frac{2}{7}$.

26. **C**. Triangle ABE is isosceles, so $\angle BAE = \angle ABE = 42^0$. $\angle BEC$ is the exterior angle of that triangle, so it is the sum of the two remote interior angles.

27. **E**. Use the formula $DE^2 = AD(CD)$. So, $8^2 = 6AD \Rightarrow AD = \frac{32}{3}$. Since CD=6, the length of AC is $\frac{14}{3}$. Therefore, the length of the radius is $\frac{7}{3}$.

28. **B**.

29. **C**. The formula for the distance between a point (a,b) and a line Ax+By+C=0 is $\frac{|Aa+Bb+C|}{\sqrt{A^2+B^2}}$. Plugging the numbers into the formula gives $\frac{|2(4)+(-3)(-7)-5|}{\sqrt{2^2+(-3)^2}}=\frac{24}{\sqrt{13}}=\frac{24\sqrt{13}}{13}$.

30. **D**. Find the length of half the chord by the Pythagorean theorem. $x^2+4^2=10^2 \Rightarrow x^2=84 \Rightarrow x=2\sqrt{21}$. Therefore, the answer is double that.