February Regional ______________ Calculus Individual Test

The answer choice E. NOTA denotes that "None of These Answers" are correct. DNE stands for "Does Not Exist". The domain and range of functions are assumed to be either the real numbers or the appropriate subset of the real numbers.

1. What is the average rate of change for the function $f(x) = \sin x$ on the interval $\left[0, \frac{3\pi}{2}\right]$?

   A) $\frac{3\pi}{2}$ B) $-\frac{2}{3\pi}$ C) $\frac{2}{3\pi}$ D) $\frac{1}{\pi}$ E) NOTA

2. Find $y'(0)$, given $y = \arcsin(x)$

   A) 1 B) -1 C) DNE D) $\sqrt{2}$ E) NOTA

3. If $f(g(x)) = g(f(x)) = x$ for all x, and $f(x) = x^3 + x^2 + 1$, find $g'(3)$.

   A) 5 B) $\frac{1}{3}$ C) $\frac{1}{5}$ D) 3 E) NOTA

4. Let R be the region bounded by the graphs $y = k^2 - x^2$ and $y = 0$ on the domain $[-k, k]$.
   Let U be the volume formed when R is rotated about the x-axis.
   Let G be the volume formed when R is rotated about the y-axis.
   For what positive value of k does $U = G$?

   A) $\frac{5}{64}$ B) 0 C) $\frac{15}{32}$ D) $-\frac{5}{64}$ E) NOTA

5. Let $f(x) = (x-1)^8$. Find the coefficient of the x^3 term in the expansion of $f'(x)$.

   A) -280 B) 280 C) 70 D) -70 E) NOTA

6. Susan needs enough titanium to build a cylindrical cage (including a floor and ceiling) around her rabid pet monkey. The problem is that sheet titanium costs $\frac{100}{\pi}$ dollars per square foot. If she needs a cage of volume $432\pi\, ft^3$, how much must she spend to minimize her cost (assume the cylinder completely encloses the monkey with no air holes or windows)?

   A) \$43,200 B) \$249,600 C) \$10,800 D) \$21,600 E) NOTA

7. Let $f(x) = x^3 - 2x^2 + 1$. Let $g(x)$ be the line that is tangent to $f(x)$ at $x = 1$. What is the x-intercept of $g(x)$?

   A) 0 B) -1 C) 1 D) 2 E) NOTA

8. Find $\frac{dy}{dx}$ if $\tan(x+y) - \tan(x-y) = 0$.

   A) 0 B) $\frac{\sec^2(x+y) + \sec^2(x-y)}{\sec^2(x+y)\sec^2(x-y)}$ C) $\frac{-\sec^2(x-y)}{2}$ D) 1 E) NOTA

9. Nathaniel decides to buy a Hummer in hopes of becoming a chick-magnet. When he leaves the dealership, he travels five kilometers up a hill at a constant $10\,{}^{km}\!/_{hr}$. When he reaches the top of the hill, he has run out of gas and begins rolling back down the hill at speed of $v(t)=5t\,{}^{km}\!/_{hr}$ until he reaches the bottom of the hill. What is his average speed, in ${}^{km}\!/_{hr}$, for the entire "trip?" (assume negligible time is spent at the top of the hill and the top of the hill represents t = 0.)

A) $\frac{40\sqrt{2}-20}{7}$ B) $\frac{40\sqrt{2}+20}{7}$ C) $\frac{20\sqrt{2}-10}{7}$ D) $\frac{20}{3}$ E) NOTA

10. $\lim\limits_{x\to\infty}\frac{\cos x}{x}$

A) 1 B) -1 C) DNE D) ∞ E) NOTA

11. For what value of k will $f(x)$ be continuous and differentiable at x=2?

$$f(x)=\begin{cases} x^2-kx, x<2 \\ k-4, x\geq 2 \end{cases}$$

A) No such k exists B) 1 C) 0 D) 3 E) NOTA

12. $\lim\limits_{x\to 0}\frac{\sin x}{x}=$

A) 0 B) 1 C) ∞ D) DNE E) NOTA

13. $\lim\limits_{x\to 3}\frac{x-3}{\sqrt{4x+3}-\sqrt{2x+9}}$

A) $\frac{\sqrt{15}}{2}$ B) $\frac{2\sqrt{15}}{3}$ C) $-2\sqrt{15}$ D) $\sqrt{15}$ E) NOTA

14. A spherical balloon has a diameter of one foot. If the diameter doubles, let:

A = The actual change (in ft^3) in the balloon's volume.

B = The change (in ft^3) in the balloon's volume using differentials for approximation.

Find $|A-B|$

A) $\frac{3\pi}{2}$ B) $-\frac{2\pi}{3}$ C) $\frac{16\pi}{3}$ D) $-\frac{16\pi}{3}$ E) NOTA

15. Let $g(x)=x^2+12x+36$, a parabola with one zero at $x=-6$. The parabola moves downward at a rate of $2\,{}^{units}\!/_{\sec}$. How fast is the distance between the zeroes changing when they are 10 units apart?

A) $\frac{4}{5}$ B) $\frac{2}{5}$ C) $\frac{1}{5}$ D) 1 E) NOTA

16. At points X and Y on the function $f(x) = x^3 - 12x$, the instantaneous rate of change of value of the function is 0. Line *m* is the tangent line at point X, and line *n* is the tangent line at point Y. What is the distance between lines *m* and *n*?
A) 16 B) 32 C) 8 D) 24 E) NOTA

17. Consider the relation $y^2 = x^2 - x$. Which of the following is equal to $\frac{d^2y}{dx^2}$?
A) $\frac{y^2 - (2x-1)^2}{4y^3}$ B) $-\frac{1}{4y^3}$ C) $\frac{4y^2 - (2x-1)^2}{4y^2}$ D) 1 E) NOTA

18. Given $f(x) = x^x$, find $f'(2)$.
A) $4(1+\ln 2)$ B) $4\ln 2$ C) 8 D) 4 E) NOTA

19. A vector-valued function in the form $r(t) = \langle x(t), y(t) \rangle$ has a derivative of $r'(t) = \langle x'(t), y'(t) \rangle$.
Given that $r(t) = \langle \sin^2 t, \sin t \cos t \rangle$, find $\left\| r'\left(\frac{2\pi}{3}\right) \right\|$, where $\|r(t)\|$ denotes the magnitude of vector $r(t)$.
A) 0 B) -1 C) 1 D) $\frac{1}{2}$ E) NOTA

20. Two identical cones (each with a small opening at their vertex) of radius $5cm$ and height $10cm$ are stacked on top of each other to form a double napped cone. Sand that filled the top cone begins falling into the empty bottom cone at a rate of $\frac{28\pi}{3}\, cm^3/\sec$, forming a perfect frustum with a larger base radius of $5cm$. At what rate is the height of the frustum increasing after seven seconds have passed?
A) $\frac{28}{27}$ B) $\frac{28\sqrt[3]{9}}{27}$ C) $7\pi\sqrt[3]{28}$ D) $28\pi\sqrt[3]{28}$ E) NOTA

21. If $\lim_{h\to 0} \frac{f(x+2h) - f(x-2h)}{4h} = 3x+2$, what is $f'(10)$?
A) 32 B) 170 C) 16 D) 30 E) NOTA

22. A figure sits on the xy-plane such that semicircular cross-sections perpendicular to the x-axis lie on the graph $4x^2 + 9y^2 = 36$ so that the endpoints of the diameter are on the ellipse. Find the surface area of the figure (including the base).
A) $3\pi^2 + 6\pi$ B) $\frac{3\pi^2}{2}$ C) $3\pi^2$ D) $6\pi^2 + 3\pi$ E) NOTA

23. Find the minimum distance between the parabola $y = x^2$ and the line $y = x - 4$.
A) $\frac{15\sqrt{2}}{8}$ B) $\frac{1}{2}$ C) $\frac{15}{4}$ D) $\frac{1}{4}$ E) NOTA

24. If a_1 is the first the first term in a geometric sequence with common ratio *r*, where $0<|r|<1$, the sum of the first *n* terms of this series is given by the formula $S_n = \frac{a_1 - a_1 r^n}{1-r}$. Find $\lim_{n\to\infty} S_n$.

A) $-\infty$ B) $\frac{a_1}{1-r}$ C) ∞ D) $a_1^{\,2} r$ E) NOTA

25. Let $f(x)=2^x - x - 1$ If $f'(4) = a\ln b - c$, find $(a-b-c)^2$.

A) 169 B) 196 C) 225 D) 256 E) NOTA

26. Evaluate, the limit: $\lim_{x\to 0}\frac{\tan x}{x}$.

A) 1 B) 0 C) DNE D) ∞ E) NOTA

27. If $\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{3x} = e^6$, then what is $\lim_{x\to\infty}\left(1-\frac{2}{x}\right)^{3x}$?

A) e^6 B) $e^{\frac{2}{3}}$ C) e^{-6} D) $e^{-\frac{3}{2}}$ E) NOTA

28. The probability of hitting a dartboard at a distance *r* from its center is proportional to $\sin r$. The probability of hitting the dartboard $\frac{\pi}{3}$ ft. from the center is $\frac{1}{2}$. Assuming that a dart will hit the dartboard every time, what is the maximum radius of the dartboard?

A) $\sin^{-1}\left(\sqrt{3}\right)$ B) $\frac{\pi}{2}$ C) $\cos^{-1}\left(\frac{3-\sqrt{3}}{3}\right)$ D) $\cos^{-1}\left(1-\sqrt{3}\right)$ E) NOTA

29. How many times to the graphs of $f(x)=x^2$, $g(x)=(x-3)^2$, and the normal line to $f(x)$ at x = 2 intersect each other? (an intersection is where *at least* two graphs intersect)

A) 1 B) 2 C) 3 D) 4 E) NOTA

Velocity (m/s)	0	10	15	25	45	50
Time (s)	0	2	4	6	8	10

30. The table above represents the velocity of an object measured at different times. Using the Left Rectangle Approximation Method with 5 subdivisions of equal width, determine the distance the object traveled between $t=0$ and $t=10$.

A) 145 m B) 95 m C) 190 m D) 290 m E) NOTA