February Regional **Calculus Team: Question #1**

If $f'(3)=5$, and $f(x)$ is continuous and differentiable everywhere, then which of the following must be true?

A) $\lim_{h\to 0}\frac{f(3+h)-f(3)}{h}=5$ B) $\lim_{x\to 3} f(x)=5$ C) $\lim_{x\to 3^+} f(x)=\lim_{x\to 3^-} f(x)$

D) $\lim_{x\to 3}\frac{f'(3+h)-f'(3)}{h}$ exists. E) $f''(3)\ge 0$ F) $f(x)$ is increasing at $x=3$.

February Regional **Calculus Team: Question #2**

Evaluate the following limits:

A) $\lim_{x\to 0}\frac{1-\cos x}{x}$ B) $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$ C) $\lim_{x\to 0}(1+x)^{1/x}$ D) $\lim_{x\to 3}\frac{x^3-3x^2+2x-6}{x-3}$

Find $\frac{C}{B}+\frac{A}{D}$

February Regional **Calculus Team: Question #3**

Evaluate the following integrals:

$A=\int_{\pi/6}^{\pi/3}\frac{1}{1-\cos x}dx$ $B=\int_0^{e^2}dx$ $C=\int_0^{\pi/3}\tan x dx$ $D=\int_0^1\frac{x^2+2x-3}{x-2}dx$

Find $\frac{A}{4}+B+5C+D$

February Regional **Calculus Team: Question #4**

Let the points (A , B) and (C , D) be the points on $y=x^2+4x+3$ closest to the point (-2, 3).

Find A + B + C + D.

February Regional **Calculus Team: Question #5**

Given the system of equations:
$$\begin{aligned}x-2y+z&=p\\2x-y-2z&=p^2\\x+2y+2z&=2p\end{aligned}$$
, what value of p will minimize the value of $x+y+z$?

February Regional **Calculus Team: Question #6**

Consider the following differential equation:

$\frac{dy}{dx}=\frac{y\ln(y+1)+\ln(y+1)}{x\ln x}$. If $y(4)=15$, then what is $y(8)$?

February Regional **-2** **Calculus Team: Question #7**

A = $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{4\ln(i+n)-4\ln n}{i+n}$ B = $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{i^2}{n(i^2+n^2)}$ $C=\lim_{n\to\infty}\sum_{i=1}^{n}\frac{i}{i^2+n^2}$

$D=\lim_{n\to\infty}\sum_{i=1}^{n}\frac{n}{i^2+n^2}$ Find $\frac{A}{C^2}+B+D$

February Regional **Calculus Team: Question #8**

Time (s)	0	1	2	3	4	5	6	7	8	9	10
Speed(m/s)	0	2	5	10	15	25	25	20	15	10	5

The table above represents the speed of an object measured at different times on the interval $0\le t\le 10$.

Use the following methods to estimate the total distance traveled by the object on the interval $0\le t\le 10$.

A = Left Rectangular Approximation Method (LRAM) using 5 rectangles of equal width.

B = RRAM using 5 rectangles of equal width.

C = Trapezoidal method using 5 rectangles of equal width.

D = MRAM using 5 rectangles of equal width.

Find A + B + C + D

February Regional **Calculus Team: Question #9**

Evaluate the following limit, given $-\frac{\pi}{2} < \arctan x < \frac{\pi}{2}$:

$$\lim_{x\to\infty} \frac{\arctan x - \frac{\pi}{2}}{\frac{1}{x}}$$

February Regional **Calculus Team: Question #10**

A) Consider a street lamp 15 feet tall and Pratik, who is 6 feet tall. Pratik starts walking away from the lamp at a constant rate of $3 \, {}^{ft}\!/_{\sec}$. At what rate (in feet per second) is the length of his shadow increasing when he is 9 feet from the lamp?

B) Consider a street lamp 16 feet tall. Kristy, standing 4 feet from the foot of the post, throws a ball directly above her head so that it reaches a max height of 16 feet. At what rate (in feet per second) is the shadow of the ball moving half a second after the ball has begun its descent? (Gravity $= -32 \, {}^{ft}\!/_{\sec^2}$)

C) Michelle is flying a kite that is 400 feet directly above her head when a very strong wind begins pulling it away (horizontally) at a constant rate of $50 \, {}^{ft}\!/_{\sec}$. How fast (in feet per second) is the kite string unraveling after 6 seconds, given that the kite does not move vertically?

Find A + B + C .

February Regional **Calculus Team: Question #11**

$$A = \int_{\pi}^{2\pi} \frac{\cos x \sin x}{\sin^2 x + \frac{1}{2}\sin 2x - x} dx \qquad B = \int_{\pi}^{2\pi} \frac{\cos^2 x}{\sin^2 x + \frac{1}{2}\sin 2x - x} dx \qquad C = \int_{\pi}^{2\pi} \frac{1}{\sin^2 x + \frac{1}{2}\sin 2x - x} dx$$

Find $A + B - C$

February Regional **Calculus Team: Question #12**

Mrs. Doker needs to make an open box to hold all her lime-green paraphernalia. To do so, she cuts congruent squares from the corners of a $9in \times 9in$ lime-green sheet of aluminum foil and then turns up the edges. In cubic inches, what is the maximum volume of Mrs. Doker's box?

February Regional **Calculus Team: Question #13**

Let A and B be two numbers between 0 and 10, inclusive, that are picked randomly and independently. In a certain game, if the product of these two numbers is greater than 50, then you win \$20. If the product is less than 50, then you lose \$6. If Sachin plays this game infinitely many times, his expected winnings/losses per play will be $a - b\ln c$, where a, b, and c are distinct prime numbers. Find a + b + c.

Expected value is $\sum_{i=1}^{n} x_i p(x_i)$, where x_i is the value of event x_i (such as \$6), and $p(x_i)$ is the probability of that event occurring.

February Regional **Calculus Team: Question #14**

Let N be the line *normal* to $f(x) = x^2 + 3x - 1$ at point R (-1, -3). Line N intersects $f(x)$ at points R and S. What is the sum of the coordinates of S?

February Regional **Calculus Team: Question #15**

Consider line L, defined by the equation $y = 2 - x$, which forms a triangle T with the positive x- and y-axes. If line L rotates clockwise about the point (1, 1) at a rate of $\frac{9}{4} \, {}^{rad}\!/_{\sec}$, how fast is the area of T increasing, in ${}^{units^2}\!/_{\sec}$ when the slope of L first equals $-\sqrt{3}$?