

- 1) The sum of the first n terms of a sequence is given by the function $S(n) = 11n - n^2$, where n is a natural number. (For example, $S(100) = 11 \cdot 100 - 100^2 = 1100 - 10000 = -8900$)

A) Let k be the number of terms of this sequence which must be added in order to obtain a sum of 0?

B) What is the maximum value, call it m of the sequence $S(n)$?

Find mk .

- 2) A quadratic function has its vertex at $(3, -4)$ and it passes through the point $\left(6, \frac{1}{2}\right)$. If the function is written in the form $y = ax^2 + bx + c$, find the value of c .

- 3) A cylindrical tank is full of water and has a drain on the bottom of the tank. The volume, $V(t)$, of the water in the tank t minutes after the drain is opened is modeled by the equation $V(t) = 10t^2 - 200t + 1000$. At what value of t (again, in minutes) will the tank be one-fourth full? (Consider only an appropriate domain and range for the circumstances of this problem. This is not reasonably defined for all real numbers.)
- 4) Ned can travel 300 kilometer per day. He leaves Bill's house and leaves behind an important package. Bill discovers this $\frac{1}{3}$ of the day after Ned leaves and he immediately sets out to catch up to Ned. As soon as Bill catches Ned, Bill delivers the package and immediately returns home with his entire journey taking $\frac{3}{4}$ of a day. Assuming Bill travels without stop, how far can he travel in one day?

- 5) If $\log 20 = x$, find
- A) The exact value of A such that $\log A = 2x$
 - B) The exact value of B such that $\log B = x + 3$
 - C) The exact value of C such that $\log C = \frac{x}{2}$

Find the exact value of $\frac{BC}{A}$

- 6) The function $f(x) = 2x^3 - 16x^2 + 38x - 24$ has three real roots, call them A , B , and C in decreasing order. Find $A - B - C$.

- 7) A) The number 365 can be expressed as a sum of squares in two different ways ($m^2 + n^2$ or $p^2 + q^2$), where m , n , p , and q are unique natural numbers. Find m , n , p , and q .
- B) The number 1729 can be expressed as a sum of cubes in two different ways ($w^3 + x^3$ or $y^3 + z^3$) where w , x , y , and z are unique natural numbers. Find w , x , y , and z .

What is the sum $m + n + p + q + w + x + y + z$?

- 8) Let A = The units digit of 2002^{2002}
Let B = The units digit of 2003^{2003}
Let C = The units digit of 2005^{2005}
Let D = The units digit of 2007^{2007}

Find $A + B + C + D$.

- 9) For $f(x) = 6x^3 + 35x^2 - 8x - 12$
Let A = The sum of the roots of the function
Let B = The product of the roots of the function
Let C = The average of the roots of the function
Let D = The greatest of the roots of the function

Find $A + B + C + D$ in lowest terms.

- 10) If $\log_B A = B$ and $\log_A B = A$, then the product $AB =$

- 11) Let W = the product of the numerator and denominator of $\sum_{n=1}^{\infty} 4 \cdot \left(\frac{-3}{8}\right)^n$ when expressed in simplified form

Let X = The coefficient of the term of the expansion of $(x - 2y)^{12}$ that contains x^9

Let Y = The maximum y coordinate of the graph of $9x^2 + 4y^2 - 54x + 16y + 61 = 0$

Let Z = The maximum x coordinate of the graph of $9x^2 + 4y^2 - 54x + 16y + 61 = 0$

Find $W+X+Y+Z$.

- 12) For $f(x) = \frac{x^2 + 5x - 4}{x - 2}$, evaluate $f(7823)$ to the nearest integer.

- 13) A tunnel is in the shape of a semi-ellipse. The tunnel is 10 feet high in the center and is 40 feet wide. What is the height of the tunnel from a point on the ground 4 feet from one of the sides of the tunnel?
- 14) A bricklayer can double his output each day on a job. If he starts with no brick laid at the beginning of day 1 and had laid a total of 5 feet of brick by the end of the fifth day, how many feet of brick did he lay the first day?

- 15) A Social Security number consists of nine digits (0 – 9). If the first three digits cannot be 000 AND the last six digits cannot be 000000 [for example 000-12-3456 is prohibited as is 345-00-0000. However, numbers like 024-09-2300 are perfectly legal.] How many possible Social Security numbers can be formed?