

1. A) For this we need to simply factor the sum function to find that the zeros occur at $n = 0$ and $n = 11$. Since n is a natural number, we use 11 for this answer.
 B) The terms of the sequence above are, in order, 10, 18, 24, 28, 30, 30, 28, 24, 18, 10, and 0. After this all terms are negative, so we see that the maximum value of the sequence is 30.

The final answer then is $11 \cdot 30 = 330$.

2. Since we are given the vertex of the function we can start with the function in vertex form. It looks like this $y = a(x-h)^2 + k \rightarrow y = a(x-3)^2 - 4$. Using the given point we can see the equation becomes

$$\frac{1}{2} = a(6-3)^2 - 4 \Rightarrow \frac{1}{2} = 9a - 4 \Rightarrow \frac{1}{2} = a.$$

Now, we can use this value of a to go back to our vertex from above and expand to find the final answer.

$$y = \frac{1}{2}(x-3)^2 - 4 \Rightarrow y = \frac{1}{2}x^2 - 3x + \frac{1}{2}, \text{ so } c = \frac{1}{2}.$$

3. Since the function describes the volume of a physical object we have a natural lower boundary of 0 for the domain of the function. The upper bound for the domain will be the capacity of the tank which is interpreted as the initial value of the function which is 1000, so **the domain is [0, 1000]**. The range is defined to be a time value in minutes after the drain is opened. The natural lower boundary is again 0 with the upper boundary determined by the time it takes to completely drain the tank. The volume function has 10 as its double root, so **the range is [0, 10]**

One-fourth of the volume is 250, so when $V(t) = 250$ we have the following quadratic to solve

$$250 = 10t^2 - 200t + 1000 \Rightarrow 0 = 10t^2 - 200t + 750 \Rightarrow 0 = t^2 - 20t + 75$$

$$0 = (t-15)(t-5) \Rightarrow t = 15, t = 5$$

Since only $t = 5$ is in our domain, that is the time when the tank is one-fourth full.

4. If Ned travels 300 kilometers per day, then he has traveled 100 kilometers before Bill discovers the package. If Bill's entire trip takes $\frac{3}{4}$ of the day, then his trip to catch up to Ned took $\frac{3}{8}$ of the day. In that amount of time Ned would have traveled $\left(\frac{3}{8} + \frac{1}{3}\right) \cdot 300 = 212.5$, so that is how far Bill traveled in $\frac{3}{8}$ of the day. So, in a whole day Bill can travel $\frac{8}{3} \cdot 212.5 = 566\frac{2}{3}$.

5. A) If $\log 20 = x$, then $2 \log 20 = 2x$ or $2 \log 20 = 2x$, so $A = 400$

B) If $\log 20 = x$, then $\log 20 + 3 = x + 3$ or $\log 20 + \log 1000 = x + 3$, so $\log 20000 = x + 3$, thus $B = 20000$

C) If $\log 20 = x$, then $\frac{1}{2} \log 20 = \frac{x}{2}$, or $\log 20^{\frac{1}{2}} = \frac{x}{2}$, so $C = \sqrt{20} \Rightarrow C = 2\sqrt{5}$

$$\text{So, } \frac{BC}{A} = \frac{20000 \cdot 2\sqrt{5}}{400} = \frac{40000\sqrt{5}}{400} = 100\sqrt{5}$$

6. $f(x) = 2x^3 - 16x^2 + 38x - 24 = 0 \Rightarrow x^3 - 8x^2 + 19x - 12 = 0$. Since the coefficients add to 0 we know that 1 is a root of this function. Synthetic division by 1 yields the following factors: $(x-1)(x^2 - 7x + 12) = 0 \Rightarrow (x-1)(x-3)(x-4) = 0$. So, the roots, in decreasing order, are $A = 4$, $B = 3$, and $C = 1$. So $A - B - C = 0$

7. A) A little smart trial and error gives us $365 = 169 + 196 = 13^2 + 14^2$, as well as $365 = 4 + 361 = 2^2 + 19^2$, so m , n , p , and q are 2, 13, 14, and 19.
B) Again, we need to rely on speed and on a smart process of trial and error to see that $1729 = 1 + 1728 = 1^3 + 12^3$ or $1729 = 1000 + 729 = 10^3 + 9^3$, so w , x , y , and z are 1, 9, 10, and 12.

The sum

$$m + n + p + q + w + x + y + z = 2 + 13 + 14 + 19 + 1 + 12 + 10 + 9 = 80$$

8. A) The pattern of units digits in the powers of 2 is 2, 4, 8, 6, 2, 4, 8, 6, ... This has a period of 4, so to find the units digit of a number like 2002^{2002} , we need to see where the number 2002 lies in the expansion pattern. 2002 is $2000 + 2$ and 2000 is divisible by 4, so 2002 is the second term in a new sequence of our pattern, so the units digit of 2002^{2002} will be 4, so $A = 4$.

B) The pattern of units digits in the powers of 3 is 3, 9, 7, 1, 3, 9, 7, 1, ... This also has a period of 4. Since $2003 = 2000 + 3$ and 2000 is divisible by 4, 2003 will be the third term in a new sequence of 4 and 2003^{2003} will have a units digit of 7.

So $B = 7$

C) All powers of 5 end in the digit 5, so $C = 5$.

D) The powers of 7 have the following pattern of 7, 9, 3, 1, 7, 9, 3, 1, ... for their units digits. Since $2007 = 2004 + 3$ so 2007 is the third term in a sequence of 4, so 2007^{2007} will have a units digit of 3, so $D = 3$.

$$A + B + C + D = 4 + 7 + 5 + 3 = 19$$

9. $f(x) = 6x^3 + 35x^2 - 8x - 12 = (3x - 2)(2x + 1)(x + 6)$ so the roots of the function are $\frac{2}{3}, -\frac{1}{2}, -6$.

$$A = \text{Sum of the roots} = \frac{2}{3} + \frac{-1}{2} + -6 = \frac{4}{6} + \frac{-3}{6} + \frac{-36}{6} = \frac{-35}{6}$$

$$B = \text{Product of the roots} = \frac{2}{3} \cdot \frac{-1}{2} \cdot -6 = \frac{-1}{3} \cdot -6 = 2$$

$$C = \text{Average of the roots} = \frac{A}{3} = \frac{-35}{18}$$

$$D = \text{The greatest of the roots} = \frac{2}{3}$$

$$A + B + C + D = \frac{-35}{6} + 2 + \frac{-35}{18} + \frac{2}{3} = \frac{-105}{18} + \frac{36}{18} + \frac{-35}{18} + \frac{12}{18} = \frac{-92}{18} = \frac{-46}{9}$$

10. If $\log_B A = B$, then $B^B = A$. If $\log_A B = A$, then $A^A = B$. Therefore,

$$(A^A)^B = B^B \Rightarrow A^{AB} = B^B \Rightarrow A^{AB} = A \Rightarrow AB = 1$$

11. W = the product of the numerator and denominator of $\frac{\frac{-3}{2}}{1 - \frac{-3}{8}} = \frac{\frac{-3}{2}}{\frac{11}{8}} = \frac{-3}{2} \cdot \frac{8}{11} = \frac{-12}{11}$,

so $W = -132$

$$X = \binom{12}{3} \cdot 1^9 \cdot (-2)^3 = 220 \cdot 1 \cdot -8 = -1760$$

For X and Y we must first convert this ellipse equation into standard form.

$$9x^2 + 4y^2 - 54x + 16y + 61 = 0 \rightarrow 9(x^2 - 6x) + 4(y^2 + 4y) = -61$$

$$9(x^2 - 6x + 9) + 4(y^2 + 4y + 4) = -61 + 81 + 16$$

$$\frac{(x-3)^2}{4} + \frac{(y+2)^2}{9} = 1$$

This ellipse has its center at (3, -2) and its horizontal axis has length 4 and its vertical axis has length 6. So,

$$Y = -2 + 3 = 1$$

$$Z = 3 + 2 = 5$$

$$W + X + Y + Z = -132 + -1760 + 1 + 5 = -1886$$

12. Synthetic division of $x^2 + 5x - 4$ by $x - 2$ yields $f(x) = \frac{x^2 + 5x - 4}{x - 2} \Rightarrow x + 7 + \frac{10}{x - 2}$, so for an x -value such as 7823, the remainder portion is so small that it will not affect the output for the function. So, $f(7823) = 7823 + 7 + \frac{10}{7823 - 2} \approx 7830$

13. For the sake of convenience, let's call the center $(0,0)$ and the equation in standard form for the function is $\frac{x^2}{20^2} + \frac{y^2}{10^2} = 1 \Rightarrow \frac{x^2}{400} + \frac{y^2}{100} = 1$. If you are 4 feet from the side of the tunnel, then $x = 16$. So, we have
- $$\frac{16^2}{400} + \frac{y^2}{100} = 1 \Rightarrow \frac{256}{400} + \frac{4y^2}{400} = \frac{400}{400} \Rightarrow 4y^2 = 144 \Rightarrow y = \pm 6.$$
- Since y is a height measure in this equation, the tunnel is 6 feet tall at this point.
14. Call the length of brick laid on the first day x . On the five days he is working he lays
- $$x + 2x + 4x + 8x + 16x = 5 \text{ ft} \Rightarrow 31x = 5 \text{ ft} \Rightarrow x = \frac{5}{31} \text{ ft}.$$
15. With no restrictions, the number of combinations would be 10^9 . Since we remove the opening 000 as a possibility, we must subtract the 10^6 combinations that would start with 000. Since the last six digits cannot be 000000, we must subtract the 10^3 combinations that end in 000000. However, we have subtracted the 000000000 combination twice now, so the final total is 998999001