**2008 Lee County Invitational** **Calculus Team: Question #1**

Use differentials to *approximate* each radical. Use the function $f(x)=\sqrt{x}$ and the given values for x and dx. Write your answers as simplified improper fractions.

(A) Approximate $\sqrt{25.5}$ using x = 25 and dx = 0.5

(B) Approximate $\sqrt{49.6}$ using x = 49 and dx = 0.6

(C) Approximate $\sqrt{81.75}$ using x = 81 and dx = 0.75

(D) Approximate $\sqrt{99.4}$ using x = 100 and dx = -0.6

**2008 Lee County Invitational** **Calculus Team: Question #2**

Find the exact value of each limit. If the limit does not exist (or approaches positive or negative infinity) write *DNE*.

(A) $\lim_{x\to 0}\dfrac{x^4+5x-3}{2-\sqrt{x^2+4}}$

(B) $\lim_{x\to 1}\dfrac{x^{1/3}-1}{x^{0.25}-x}$

(C) $\lim_{x\to -2}\dfrac{\frac{1}{x}+\frac{1}{2}}{x^3+8}$

(D) $\lim_{x\to \frac{\pi}{4}}\dfrac{1-\tan x}{\sin x-\cos x}$

**2008 Lee County Invitational** **Calculus Team: Question #3**

Find each sum.

(A) $\sum_{i=1}^{18}(i^2+4)$

(B) $\sum_{i=4}^{15}(2i-3)$

(C) $\sum_{i=1}^{14}(i^3+i)$

(D) $\sum_{i=1}^{10}(i-1)^2$

**2008 Lee County Invitational** **Calculus Team: Question #4**

Find the *average value* of each function on the given interval.

(A) $f(x)=x-2\sqrt{x}$ on the interval [0, 4]

(B) $f(x)=x^2-4$ on the interval [0, 5]

(C) $f(x)=\frac{2}{x}$ on the interval [1, 8] [Answer *must be* in terms of ln(2)]

(D) $f(x)=\cos x-\sin x$ on the interval [0, $\frac{\pi}{6}$]

**2008 Lee County Invitational** **Calculus Team: Question #5**

$$h(x) = f(x)g(x) \text{ and } p(x) = \frac{f(x)}{g(x)}$$

Use the table below to find the exact values of the derivatives at the given points. Write your answers in *simplified fraction form*.

| | $x = 1$ | $x = 2$ |
|---|---|---|
| $f(x)$ | 4 | 6 |
| $g(x)$ | 1/3 | 1/2 |
| $f'(x)$ | 1/4 | 4 |
| $g'(x)$ | -8 | 12 |
| $f''(x)$ | -3/2 | -1 |
| $g''(x)$ | 10 | -2 |

(A) $h'(2) = ?$ (B) $p'(1) = ?$ (C) $p'(2) = ?$ (D) $h''(1) = ?$

**2008 Lee County Invitational** **Calculus Team: Question #6**

For each part, find the value(s) of c guaranteed by the indicated theorem. If the stated theorem does not apply, write "*does not apply*".

(A) Find all values of "c" that satisfy Rolle's Theorem for $f(x) = x^4 - 2x^2$ on the interval [-2, 2].

(B) Find all values of "c" that satisfy Rolle's Theorem for $f(x) = x - x^{\frac{1}{3}}$ on the interval [-1, 1].

(C) Find all values of "c" that satisfy the Mean Value Theorem for derivatives for $f(x) = x^2$ on the interval [-4, 1].

(D) Find all values of "c" that satisfy the Mean Value Theorem for derivatives for $f(x) = x(x^2 - 3x - 4)$ on the interval [-1, 1].

**2008 Lee County Invitational** **Calculus Team: Question #7**

Given the function $f(x) = x^4 - 4x^3$,

(A) On what interval(s) is $f(x)$ increasing?

(B) On what interval(s) is $f(x)$ decreasing?

(C) On what interval(s) is $f(x)$ concave upward?

(D) On what interval(s) is $f(x)$ concave downward?

**2008 Lee County Invitational** **Calculus Team: Question #8**

Find the exact value of each definite integral.

(A) $\int_0^2 |x-2|\,dx$

(B) $\int_0^4 |2x-3|\,dx$

(C) $\int_0^5 |8-2x|\,dx$

(D) $\int_2^6 |4-x|\,dx$

**2008 Lee County Invitational** **Calculus Team: Question #9**

For each of the following, find two positive numbers A and B that satisfy the given requirements. *All radicals must be in simplest form* and answers for each part should be given in the form (A, B).

(A) The product is 192 and the sum is a minimum.

(B) The product is 192 and the sum of the first and three times the second is a minimum, where A is the first number and B is the second number.

(C) The product is 108 and the sum is a minimum.

(D) The product is 108 and the sum of the first and three times the second is a minimum, where A is the first number and B is the second number.

**2008 Lee County Invitational** **Calculus Team: Question #10**

Given the curve: $3xy^2 + 2x^2y + 4y = xy$,

(A) Find the slope of the tangent to the curve at the point (0, 0).

(B) Find the slope of the tangent to the curve at the point (1, -5/3)

(C) Find the slope of the normal to the curve at the point (1, -5/3)

(D) Find the slope of the tangent to the curve at the point (-1, 7/3)

**2008 Lee County Invitational** **Calculus Team: Question #11**

For each part, find the *exact value*.

(A) If $f(x) = 12\sec x$, then $f'(\frac{7\pi}{6}) = ?$

(B) If $f(x) = -3\csc x$, then $f'(\frac{14\pi}{3}) = ?$

(C) If $f(x) = 4\sin x$, then $f'(\frac{5\pi}{12}) = ?$

(D) If $f(x) = 20\cos x$, then $f'(\frac{\pi}{12}) = ?$

**2008 Lee County Invitational** **Calculus Team: Question #12**

For each part, find the *exact value* of the definite integral.

(A) $\int_1^5 3^x dx = ?$

(B) $\int_{1/2}^{5/2} \frac{x}{\sqrt{2x-1}} dx = ?$

(C) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (\cos x - 2\sec^2 x) dx = ?$

(D) $\int_0^4 4xe^x dx = ?$

**2008 Lee County Invitational** **Calculus Team: Question #13**

If $f(x) = 9x^5 + \ln(x) - \cos x$, then

(A) $f'(x) = ?$

(B) $f''(x) = ?$

(C) $f^{(3)}(x) = ?$

(D) $f^{(4)}(x) = ?$

**2008 Lee County Invitational** **Calculus Team: Question #14**

Given that ln(2) = 0.69 and ln(5) = 1.61, use the properties of logarithms to approximate each of the following to two decimal places.

(A) ln(20) = ?

(B) $\ln(\frac{5}{2}) = ?$

(C) $\ln(\frac{1}{40}) = ?$

(D) $\ln(\sqrt[3]{200}) = ?$

**2008 Lee County Invitational** **Calculus Team: Question #15**

For each of the following, find *all* points of inflection.

(A) $f(x) = x^3 - 6x^2 + 12x$

(B) $f(x) = 2x^4 - 8x + 3$

(C) $f(x) = 6x^4 - 9x^3$

(D) $f(x) = \dfrac{x+1}{\sqrt{x}}$