1. Notice that, for example, 6 games to be played with the Yankees winning the first 5 games will have 3 Yankee wins and 2 Mets wins in any order, but the sixth game MUST have a Yankee win

   So now P(end without 7 games) = 1 – P(end with 7 games)
   = 1 – [P(Yankees win in 7) + P(Mets win in 7)]
   $= 1-[{}_6C_3(\frac{9}{10})^3(\frac{1}{10})^3 * (\frac{9}{10}) + {}_6C_3(\frac{9}{10})^3(\frac{1}{10})^3 * (\frac{1}{10})] = 0.9854$

2. There are 4 prime numbers in each suit, so the probability of drawing a prime is $\frac{16}{52}$. Also, there are 26 red cards, so the probability of drawing red is $\frac{26}{52}$. Now, there are 8 prime, red cards, so that probability is $\frac{8}{52}$. So,
   P(red or prime) = $\frac{16}{52}+\frac{26}{52}-\frac{8}{52}=\frac{17}{26}$.

3. By definition $aX+b$, has mean $a\mu+b$. So the mean is $-2\mu-7$.

4. By definition $aX+b$ has variance $a^2\sigma^2$. So the variance is $4\sigma^2$, so the standard deviation is 2σ.

5. In this problem $\hat{\pi}=\frac{10}{96}$. Also, $Z_{\frac{.10}{2}}=Z_{.05}=1.645$ from the table at the top of the test. So, the confidence interval is
   $$(\frac{10}{96}-1.645\sqrt{\frac{(\frac{10}{96})(\frac{86}{96})}{96}},\frac{10}{96}+1.645\sqrt{\frac{(\frac{10}{96})(\frac{86}{96})}{96}})\equiv(0.0529,0.1554)$$

6. First, by Heron's Formula (with $s=\frac{5+6+9}{2}=10$), the area of the triangle is $\sqrt{10(10-5)(10-6)(10-9)}=10\sqrt{2}$. Now the radius of the inscribed circle can be found by $r=\frac{\text{area}\Delta}{\text{semiperimeter}}=\frac{10\sqrt{2}}{10}=\sqrt{2}$. So the area of the circle is $(\sqrt{2})^2\pi=2\pi$. So the probability it lands inside the triangle but outside the circle is
   $\frac{10\sqrt{2}-2\pi}{10\sqrt{2}}=\frac{20-2\sqrt{2}\pi}{20}=\frac{10-\sqrt{2}\pi}{10}$

7. Notice that
   $$P(4\le X\le 16)=P(|X-10|\le 6)=P(|X-10|\le 4(\frac{3}{2}))=P(|X-10|\le\frac{3}{2}\sigma)\ge 1-\frac{1}{(3/2)^2}$$

So the probability is at least 5/9.

8. Since you are trying to refute the claim that no more than 10 sales a week are being made, that is the alternative hypothesis. Thus, H_0: $\mu \le 10$, H_a: $\mu > 10$.

9. The test statistic for the study is $\frac{12-10}{7/\sqrt{49}} = 2$. So the p-value is $P(Z \ge 2) = .0228$

10. Type II

11. The spread of the middle 50% of the data is the interquartile range (IQR) of the data. Rearrange the data in ascending order:
-12, -8, 3, 4, 5, 6, 9, 9, 9, 10, 10, 10. Now the median of the data is between 6 and 9. So $Q_1 = \frac{3+4}{2} = 3.5$ and $Q_3 = \frac{9+10}{2} = 9.5$. So, IQR = 9.5 – 3.5 = 6.

12. The Law of Large numbers is a theorem that describes the long-run stability of a random variable. Thus, in the long run a coin will come up heads $\frac{1}{2}$ of the time.

13. Using the formula
$\sigma^2 = E(X^2) - [E(X)]^2 \Rightarrow 7^2 = 55 - [E(X)]^2 \Rightarrow [E(X)]^2 = 6 \Rightarrow E(X) = \sqrt{6}$

14. Using the left endpoint, the formula is $\frac{100}{400} - Z_{a/2}\sqrt{\frac{.25(1-.25)}{400}} = 0.218$. Solving yields that $Z_{a/2}$ = 1.478. So a/2 is between .0694 and .0708. So a is between 0.1388 and 0.1416. So a falls in (.12,.16).

15. A number has an odd number of factors if it is a perfect square (for example, 4 has the factors of 1, 2 and 4). The first 100 whole numbers are {0,1,2,…,99}. So the numbers with an odd number of factors are 1, 4, 9, 16, 25, 36, 49, 64 and 81 (0 has an infinite number of factors). So the probability is $\frac{9}{100}$.

16. The Z-score associated with a height of 68 is $\frac{68-62}{4} = 1.5$. So
$P(X \ge 68) = P(Z \ge 1.5) = .0668$

17.

18. Since the probabilities must sum to 1: $.46 + .23 + X + .03 + .01 = 1 \Rightarrow X = .27$

19. To get the mean we take a weighted average:

$$\frac{0(0.46)+1(0.23)+2(0.27)+3(0.03)+4(0.01)}{0.46+0.23+0.27+0.03+0.01}=0.9$$

20. The median is the number which 50% of the data falls below. Since 1 has 69% of the data at, or below, it, we conclude that 1 is the median.

21. P(more men than women|at least 1 woman) =

$$\frac{P(\text{moremen} \cap \text{atleast1woman})}{P(\text{atleast1woman})}=\frac{P(\text{moremen} \cap \text{atleast1woman})}{1-P(\text{nowomen})}$$

$$=\frac{\frac{({}_{10}C_4)({}_8C_1)}{{}_{18}C_5}+\frac{({}_{10}C_3)({}_8C_2)}{{}_{18}C_5}}{1-\frac{{}_{10}C_5}{{}_{18}C_5}}=\frac{20}{33}$$

22. I only (Skewed right would have a heavy low end tail, normal would be symmetric and mound shaped and bimodal would have 2 extremes)

23. First, the mean wingspan is $\frac{3+9+1+5+5+10+3+6+5}{9}=\frac{47}{9}$. Now the maximum magnitude for sampling error will occur if we select the 3 highest values or the 3 lowest values. Now the mean when selecting the three highest values is $\frac{6+9+10}{3}=\frac{25}{3}$ and the magnitude of the sampling error is $|\frac{47}{9}-\frac{25}{3}|=\frac{28}{9}$. The mean when selecting the three lowest values is $\frac{1+3+3}{3}=\frac{7}{3}$ and the magnitude of the sampling error is $|\frac{47}{9}-\frac{7}{3}|=\frac{26}{9}$. So the maximum magnitude of the sampling error is $\frac{28}{9}$.

24. By definition, t-distribution.

25. The test statistic is $t=\frac{2.7-3.2}{\sqrt{\frac{1.6^2}{36}+\frac{1.4^2}{42}}}=-1.46$. So the p-value is $P(Z \le -1.46)=0.07$ to 2 decimals.

26. By definition, stratified random sample

27. The correlation coefficient, r, is the square root of the coefficient of determination and has the same sign (positive or negative) as the least-squares line. So $r = \sqrt{r^2} = \sqrt{0.8843} = \pm 0.9404$, but since the slope of the least squares line is negative r = -0.9404.

28. The sum of the residuals to a line of best fit is always 0. So if the sum of the first 30 residuals is 4.5, the sum of the remaining 4 must be -4.5 so that the sum of all of the residuals is equal to 0.

29. I only (II should say $\overline{Y}$ looks more normal as the sample size increases, and III should say $\overline{Y}$ has standard deviation of $\frac{\sigma}{\sqrt{n}}$)

30. Using algebra we can show that $(1-\frac{\alpha}{2}) > (1-\alpha)$ for all values of α between 0 and 1. We know that as the confidence level of the interval is increased, the width of the confidence interval decreases. Thus, X’<X and Y’>Y

Answers Statistics Individual – January 2008 Invitational

1. D
2. B
3. A
4. E
5. B
6. C
7. C
8. A
9. B
10. B
11. B
12. D
13. E
14. C
15. A
16. E
17. C

18. A
19. B
20. B
21. D
22. A
23. D
24. B
25. C
26. C
27. A
28. D
29. A
30. D