

For each of the following questions mark the best answer on your scantron. If the correct answer is not present, choose choice E for "None of the above."

- What is the range of the function  $y=4\cos(2x + 7) - 3$   
A.  $[-4,4]$  B.  $[-4,-2]$  C.  $[-7,1]$  D.  $[3,11]$  E. NOTA
- A circle with circumference  $6\pi$  inches is the great circle for a sphere. Half the volume of the sphere is the volume of a right, circular cone with radius of 7 inches. What is the height of the cone, in inches?  
A.  $\frac{54}{49}$  B.  $\frac{18}{49}$  C.  $\frac{72}{49}$  D.  $\frac{1728}{49}$  E. NOTA
- If  $\csc(x) = \frac{11}{5}$  for an angle  $x$  in quadrant II, what is  $\cot(x)$ ?  
A.  $\frac{5\sqrt{6}}{24}$  B.  $\frac{4\sqrt{6}}{5}$  C.  $\frac{-5\sqrt{6}}{24}$  D.  $\frac{-4\sqrt{6}}{5}$  E. NOTA
- Which of the following accurately describes the following statement: "x is at least 5 units away from 4."  
A.  $|x-4| \geq 5$  B.  $|x-5| \geq 4$  C.  $|x-4| > 5$  D.  $|x-5| > 4$  E. NOTA
- If  $f(x)$  is a continuous, odd function,  $g(x)$  is a continuous, even function and  $h(x)$  is a continuous function that is neither even nor odd, then how many of the following must be even functions?  
 $[h(x)]^2$   $g(x)-f(x)$   $g(f(x))$   $h(g(x))$   $\cos(f(x))$   $\sin(g(x))$   $f(x)g(x)h(x)$   
A. 2 B. 3 C. 4 D. 5 E. NOTA
- What is the tangent of the acute angle formed by the intersection of the vectors  $(2,5,-1)$  and  $(-1,3,3)$ ?  
A.  $\frac{\sqrt{570}}{57}$  B.  $\frac{\sqrt{570}}{10}$  C.  $\frac{\sqrt{470}}{10}$  D.  $\frac{\sqrt{470}}{47}$  E. NOTA

7. What is the sum of the real solutions of the following equation:  $9^x + 3^{(x+1)} - 18 = 0$ ?
- A. 3      B. 1      C. -2      D. -3      E. NOTA
8. If you are dealt five cards from a standard deck of 52 cards, what is the probability of being dealt a four of a kind, given that one of the cards dealt to you is a four?
- A.  $\frac{624}{52C_5}$       B.  $\frac{1248}{52C_5}$       C.  $\frac{2496}{52C_5}$       D.  $\frac{4992}{52C_5}$       E. NOTA
9. This Scottish mathematician, born in 1550, is best known for the creation of logarithms whose discussion first appeared in *Mirifii logarithmorum canonicis descriptio* in 1614. Who is he?
- A. Leonhard Euler      B. Carl Gauss      C. John Napier  
D. Fred Logarithm      E. NOTA
10. A set is said to be *closed* under a certain operation if, when you take any two elements from the set and perform the operation on them, the resulting answer is also in the set. For example, the set of integers under addition is closed because the sum of any two integers is an integer. Which of the following sets are also closed under the given operation?
- A. Irrational Numbers, Multiplication      B. Natural Numbers, Subtraction  
C. Integers, Division      D. Real Numbers, Division      E. NOTA
11. Solve for  $x$  over the real numbers:  $2\tan\left(\frac{x}{2}\right) - 1 = 1$
- A.  $\pi + \pi \cdot k, k \in \{\text{integers}\}$       B.  $\pi + 2\pi \cdot k, k \in \{\text{integers}\}$   
C.  $\frac{\pi}{2} + \pi \cdot k, k \in \{\text{integers}\}$       D.  $\frac{\pi}{2} + 2\pi \cdot k, k \in \{\text{integers}\}$       E. NOTA
12. Which of the following is/are true:
- I.  $i^{47} = -i$  where  $i = \sqrt{-1}$   
II.  $[\text{cis}(\theta)]^n = \text{cis}(\theta^n)$  where  $\text{cis}(\theta) = \cos(\theta) + i\sin(\theta)$  and  $i = \sqrt{-1}$   
III. 17 is a complex number.
- A. I only      B. II only      C. I and III only      D. II and III only      E. NOTA
13. An equation of the form  $x^3 + 3x^2 + Ax - 24 = y$  has three, distinct integral roots that form an arithmetic sequence. What is A?
- A. 22      B. -22      C. 10      D. -10      E. NOTA

14. Evaluate:  $\sin(2\text{Sec}^{-1}(\frac{7}{3}))$

- A.  $\frac{12\sqrt{10}}{49}$    B.  $\frac{4\sqrt{10}}{3}$    C.  $\frac{-31}{7}$    D.  $\frac{31}{7}$    E. NOTA

15. For the circle defined by the equation  $x^2 + y^2 - 10x + 2y - 6 = 0$ , what is the slope of the line tangent to the circle at the point (1,3)?

- A.  $\frac{1}{2}$    B.  $\frac{-1}{2}$    C. 1   D. -1   E. NOTA

16. Mike is on "Press Your Luck." After the question round he heads to the "Big Board" with only 1 spin. However, Mike has done his research and he knows that on any spin of the big board he can pick up a bonus spin with probability of  $\frac{1}{5}$ . What is the total expected number of spins Mike will have at the Big Board?

- A.  $\frac{6}{5}$    B.  $\frac{15}{7}$    C.  $\frac{5}{4}$    D.  $\frac{25}{16}$    E. NOTA

17. Solve for x:  $\sin(x) - \sin^2(x) + \sin^3(x) - \sin^4(x) + \dots = \frac{1}{2}$  where  $\sin(x)$  does not equal 1 or -1.

- A.  $\frac{\pi}{2}$    B.  $\frac{-\pi}{2}$    C.  $\frac{\pi}{4}$    D.  $\frac{-\pi}{4}$    E. NOTA

18. If  $\cos(4x) = \frac{3}{8}$  and x is a Quadrant I angle, then what is  $[\sin(2x)]^2$ ?

- A.  $\frac{5}{16}$    B.  $\frac{495}{1024}$    C.  $\frac{\sqrt{55}}{8}$    D.  $\frac{55}{64}$    E. NOTA

**Questions 19 and 20 refer to the following equation:  $9x^2 - 4y^2 - 72x - 48y - 36 = 0$**

19. What are the coordinates of the center of the hyperbola?

- A. (-2, 3)   B. (4, -6)   C. (2, -3)   D. (-4, 6)   E. NOTA

20. What are the equations of the asymptotes of the hyperbola?

A.  $y = \frac{2}{3}x - 12$     B.  $y = \frac{-2}{3}x + 6$     C.  $y = \frac{-3}{2}x + 6$     D.  $y = \frac{3}{2}x - 12$     E. NOTA

$y = \frac{-2}{3}x$                        $y = \frac{2}{3}x$                        $y = \frac{3}{2}x$                        $y = \frac{-3}{2}x$

21. In triangle ABC,  $m\angle C = 30^\circ$ ,  $c=2$ ,  $\frac{\sin(A)}{\sin(B)} = \sqrt{3}$  and the area of the triangle is  $\sqrt{3}$ . What is the perimeter of this triangle? (NOTE: Capital letters represent angles and lowercase letters represent side lengths)

A.  $2 + \sqrt{3}$     B.  $8\sqrt{3}$     C.  $4 + 2\sqrt{3}$     D.  $2 + 2\sqrt{3}$     E. NOTA

22. According to the Rational Root Theorem, what are all possible rational roots of the equation defined by  $y = 100x^{100} + 99x^{99} + \dots + 2x^2 + x + 1$ ?

A. 1, -1    B. 100, -100    C.  $\frac{1}{100}, \frac{-1}{100}$     D. 0    E. NOTA

23. Simplify the following expression for natural numbers  $n > 3$ :  $\frac{(n+2)!(nP_4)}{(nC_4)(nP_{(n-1)})}$

A.  $4n^2 + 12n + 8$     B.  $n^2 + 3n + 2$     C.  $(n+1)!(n^2 + 3n + 2)$     D.  $24n^2 + 72n + 48$     E. NOTA

24. For two non-empty sets A and B, simplify the following:  $(A \cap B)' \cap (B \cup A')$  (NOTE: A' denotes the complement of set A)

A. B    B.  $A \cap B'$     C. A'    D.  $A' \cup B'$     E. NOTA

25. Where  $\lceil x \rceil$  is the greatest integer function of x, evaluate the following:

$$\lceil \cos^{-1}(-1) \rceil + \lceil -e \rceil + \lceil 1.4 \rceil + \left\lceil \log \frac{1}{2000} \right\rceil$$

A. -3    B. -2    C. -1    D. 0    E. NOTA

26. Find the volume of the solid which results when the region bounded by the graphs  $y = 0$ ,  $x = -3$ ,  $x = 3$  and  $y = |x|$  is revolved about the y-axis.

A.  $9\pi$     B.  $18\pi$     C.  $27\pi$     D.  $36\pi$     E. NOTA

27. Spurred by his success as the host of “Survivor”, Jeff Probst opens a wilderness store specializing in exotic merchandise. One of Jeff’s favorite items is his collection of “Survivor Tiki Torches” which he produces himself! Jeff has noticed that when the price of his torches is \$10 he sells 400 of them. When he raises the price of the torches by \$2 his sales fall by 40 torches. The relationship between the torches sold,  $x$ , and price of the torch,  $p$ , is linear. What is the equation expressing  $p$  as a function of  $x$ ?

- A.  $p = \frac{-1}{20}x + 30$       B.  $p = -20x + 10010$       C.  $p = \frac{1}{20}x + 30$   
 D.  $p = 20x + 10010$       E. NOTA

28. Given the function  $y = f(x)$ , which of the following represents the graph of translating  $f(x)$  to the right 1 unit and then reflecting it about the line  $y = x$ ?

- A.  $y = f^{-1}(x-1)$     B.  $y = f^{-1}(x) + 1$     C.  $y = f^{-1}(x+1)$     D.  $y = f^{-1}(x) - 1$     E. NOTA

29. Solve the following equation for  $x$ :  $12x^2 + 14x - 28y - 24xy = 0$

- A.  $\{\frac{7}{6}, 2y\}$     B.  $\{\frac{-7}{6}, -2y\}$     C.  $\{\frac{7}{6}, -2y\}$     D.  $\{\frac{-7}{6}, 2y\}$     E. NOTA

30. Assume the following information:  
 All hippogriffs are thestrals  
 All thestrals are grindylows  
 Some grindylows are pygmy puffs  
 No pygmy puffs are hippogriffs

Which of the following are invalid conclusions?

- I. All hippogriffs are grindylows  
 II. No thestrals are pygmy puffs  
 III. There is a pygmy puff that is both a thestral and a grindylow

- A. I and II      B. II and III      C. II only      D. I, II and III      E. NOTA