SOUTHWEST FLORIDA INVITATIONAL

Geometry Team Solutions

January 26, 2008

1) A.

$x^2 + x^2 = \left(\sqrt{6}\right)^2$

$2x^2 = 6$

$x = \sqrt{3}$

B.

$x^2 + 9^2 = (x+4)^2$

$x^2 + 81 = x^2 + 8x + 16$

$8x = 65$

$x = \frac{65}{8} = 8\frac{1}{8} = 8.125$

C.

$1^2 + x^2 = \left(\sqrt{3^2 + 2^2}\right)^2$

$1 + x^2 = 13$

$x^2 = 12$

$x = 2\sqrt{3}$

D.

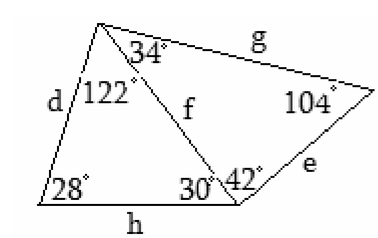
$8^2 + a^2 = 17^2$

$a = 15$

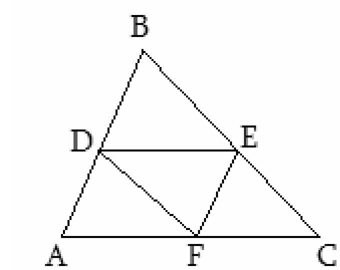
9-12-15 is a right triangle, so x=45.

2) The smallest side of the triangle on the left is the largest side of the triangle on the right. So, from largest to smallest, the sides go h, d, f, g, e.

A. h
B. d
C. g
D. e



3) $\overline{AB} = 2\overline{EF}$, $\overline{BC} = 2\overline{DF}$, $\overline{AC} = 2\overline{DE}$



A.

$\overline{BC} = 2\overline{DF}$

$4x - 2 = 2(x+3)$

$2x = 8$

$x = 4$

B&C.

$AB = 2EF$

$\frac{3}{2}x + 2y = 2(y + 3z)$

$\frac{3}{2}(4) + 2y = 2y + 6z$

$6 = 6z$

$z = 1$

$AC = 2DE$

$2x - 2z = 2(-3x + 3y)$

$2x - 2z = -6x + 6y$

$2(4) - 2(1) = -6(4) + 6y$

$8 - 2 = -24 + 6y$

$6y = 30$

$y = 5$

D.

$P = -3x + 3y + y + 3z + x + 3$

$P = -2x + 4y + 3z + 3$

$P = -8 + 20 + 3 + 3$

$P = 18$

4) A.

$slope = \frac{11 - (-4)}{6 - 3} = 5$

$5(x - 3) = y - (-4)$

$5x - 15 = y + 4$

$5x - y = 19$

$5 - 1 + 19 = 23$

B.

$3x + 6y = 17$

$6y = -3x + 17$

$y = \frac{-1}{2}x + \frac{17}{6}$

$slope = \frac{-1}{2}$

$\frac{-1}{2}(x - (-4)) = y - 5$

$x + 4 = -2(y - 5)$

$x + 4 = -2y + 10$

$x + 2y = 6$

$1 + 2 + 6 = 9$

C.

$$2x-4y=7$$
$$4y=2x-7$$
$$\frac{1}{2}x-\frac{7}{4}=y$$
$$slope=\frac{1}{2}$$
$$PerpSlope=-2$$
$$-2(x-2)=y-(-7)$$
$$-2x+4=y+7$$
$$-2x-y=3$$
$$2x+y=-3$$
$$2+1-3=0$$

D.

$$Slope=\frac{20-6}{0-1}=-14$$
$$PerpSlope=\frac{1}{14}$$
$$Midpt=\left(\frac{1+0}{2},\frac{6+20}{2}\right)=\left(\frac{1}{2},13\right)$$
$$\frac{1}{14}\left(x-\frac{1}{2}\right)=y-13$$
$$x-\frac{1}{2}=14y-182$$
$$2x-1=28y-364$$
$$2x-28y=-363$$
$$2-28-363=-389$$

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5) A.

$$\left(\frac{2\sqrt{5}}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2=s^2$$

$$5+\frac{7}{4}=s^2$$

$$\frac{27}{4}=s^2$$

$$s=\frac{3\sqrt{3}}{2}$$

$$P=4s=6\sqrt{3}$$

B.

$P=6,$ so $semiperimeter=3$

$Length=L$, $Width=3-L$

$$L^2+(3-L)^2=\left(\sqrt{5}\right)^2$$

$$2L^2-6L+4=0$$

$$L^2-3L+2=0$$

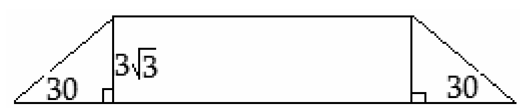
$$(L-2)(L-1)=0$$

$$L=2,1$$

The length is 2 and the width is 1, so the length of the longer side is 2.

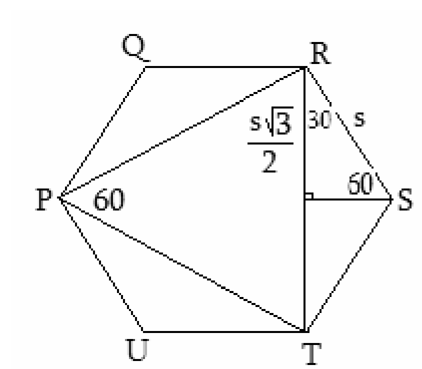
C.

Both side triangles are 30-60-90 triangles, where the side across from the 30 degree angle measure is $3\sqrt{3}$, which means the leg is $6\sqrt{3}$. The sum of the bases is two times the length of the median, or 20. So the perimeter is $20+12\sqrt{3}$.



D.

Each side of triangle PRT is going to be $\sqrt{3}$ times the length of the side of hexagon PQRSTU. So the ratio of the perimeter of the triangle to the perimeter of the hexagon is $3\sqrt{3}:6$, or $\sqrt{3}:2$.



$$\frac{\sqrt{3}}{3}=\frac{2}{P_{HEX}}$$

$$P_{HEX}=2\sqrt{3}$$

6) A.

$$\frac{OT}{OG} = \frac{AT}{AG}$$

$$\frac{5}{10/3} = \frac{AT}{26/3}$$

$$AT = 13$$

B.

With sides of length 5, 12, and 13, triangle OAT is a right triangle! $\angle GOT = 90^\circ$

C.

$$\cos(\angle OAT) = \frac{adjacent}{hypotenuse} = \frac{12}{13}$$

D.

$$5^2 + \left(\frac{10}{3}\right)^2 = (GT)^2$$

$$25 + \frac{100}{9} = (GT)^2$$

$$\frac{325}{9} = (GT)^2$$

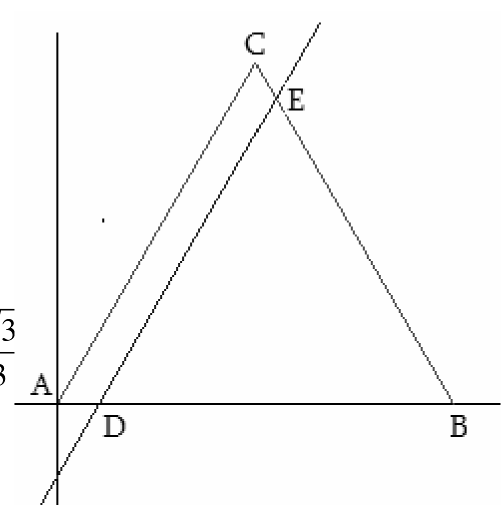
$$GT = \frac{5\sqrt{13}}{3}$$

A.

shortest distance is $\left|\frac{(\sqrt{3})(0)+(-1)(0)-1}{\sqrt{(\sqrt{3})^2+(-1)^2}}\right| = \frac{1}{2}$

B.

$DE = DB = AB - AD = \frac{18-\sqrt{3}}{3}$ sinceAB=6 and AD= $\frac{\sqrt{3}}{3}$



C.

$$D\left(\frac{\sqrt{3}}{3},0\right).$$

E is the vertex at the height of triangle DEB. So,

$$s = xvalue_{Midpt-\overline{DB}} = \frac{6-\frac{\sqrt{3}}{3}}{2} = \frac{18-\sqrt{3}}{6} + \frac{\sqrt{3}}{3} = \frac{18+\sqrt{3}}{6}$$

$$t = h_{\triangle DEB} = side \cdot \sqrt{3} = \frac{\sqrt{3}\left(18-\sqrt{3}\right)}{6} = \frac{18\sqrt{3}-3}{6}$$

$$q+r+s+t = \frac{\sqrt{3}}{3} + 0 + \frac{18+\sqrt{3}}{6} + \frac{18\sqrt{3}-3}{6}$$

$$q+r+s+t = \frac{2\sqrt{3}+18+\sqrt{3}+18\sqrt{3}-3}{6}$$

$$q+r+s+t = \frac{15+21\sqrt{3}}{6} = \frac{5+7\sqrt{3}}{2}$$

D.

$$\frac{Perimeter(\triangle BED)}{Perimeter(\triangle ABC)} = \frac{3\left(\frac{18-\sqrt{3}}{3}\right)}{3(6)}$$

$$\frac{Perimeter(\triangle BED)}{Perimeter(\triangle ABC)} = \frac{18-\sqrt{3}}{18}$$

8) A.

MO is half of the length of segment IJ in triangle IJL, since MO is parallel to IJ and because MO lies on the median. So, MO=3.

B.

Since the trapezoid is isosceles, MO=PN=3. MN, the median, has a length of 11. So,

$$OP = MN - MO - PN$$

$$OP = 11-6$$

$$OP = 5$$

C.

Draw an auxiliary line perpendicular to the lower base from the top base through P. This imaginary line hits the upper base at A, and the lower base at B.

$$JA = \frac{IJ - OP}{2} = \frac{1}{2}$$

$$(JA)^2 + (AP)^2 = (JP)^2$$

$$\left(\frac{1}{2}\right)^2 + \left(\frac{5\sqrt{3}}{2}\right)^2 = (JP)^2$$

$$\frac{1}{4} + \frac{75}{4} = (JP)^2$$

$$(JP)^2 = 19$$

$$JP = \sqrt{19}$$

D.

$$KB = \frac{LK - OP}{2} = \frac{11}{2}$$

$$(KB)^2 + (BP)^2 = (PK)^2$$

$$\left(\frac{11}{2}\right)^2 + \left(\frac{5\sqrt{3}}{2}\right)^2 = (PK)^2$$

$$(PK)^2 = \frac{121}{4} + \frac{75}{4}$$

$$(PK)^2 = \frac{196}{4}$$

$$PK = 7$$

9)

A.

$$180 - (90 - x) = 7x$$

$$90 + x = 7x$$

$$90 = 6x$$

$$x = 15$$

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B.

$n + 36n - 21 = 90$

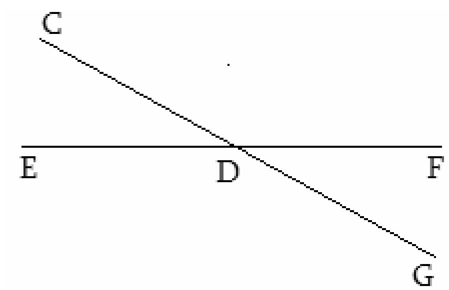
$37n - 21 = 90$

$37n = 111$

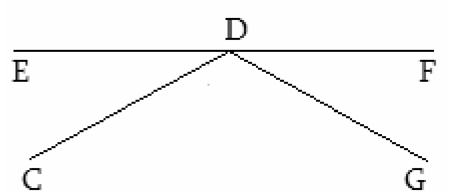
$n = 3$

C.

There are three possible scenarios, and all are shown below.



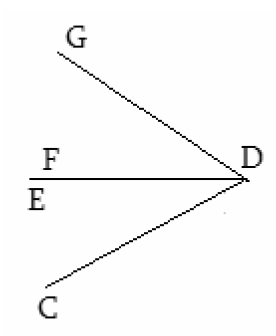
$m\angle CDG = 180$



$m\angle CDG = 180 - m\angle CDE - m\angle FDG$

$m\angle CDG = 180 - 40 - 40$

$m\angle CDG = 100$



$m\angle CDG = m\angle CDE + m\angle FDG$

$m\angle CDG = 40 + 40$

$m\angle CDG = 80$

100+180+80=360

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D.

The angles of a triangle add up to 180 degrees, so $x+(x+y)+(x+5y)=180$ or $3x+6y=180$. Also, the largest angle in the triangle is 90 degrees. Since x and y are both positive integers, then the angle with measure $(x+5y)^\circ$ will be the largest. So, $x+5y=90$.

So, it's a system of equations.

$3x+6y=180$

$x+5y=90 \rightarrow x=90-5y$

$3(90-5y)+6y=180$

$270-15y+6y=180$

$9y=90$

$y=10$

$x+5(10)=90$

$x+50=90$

$x=40$

$\frac{x}{y}=\frac{40}{10}=4$

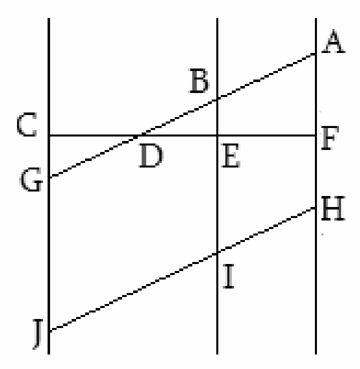
10) A.

$m\angle BDE=30$

$m\angle DBE=60$

$m\angle EIH=60$

$m\angle EIJ=180-\angle EIH=120$



B.

$\frac{BE}{AF}=\frac{DB}{DA}$

$DB=2\cdot BE \qquad (30,60,90)$

$\frac{3}{4}=\frac{6}{DA}$

$DA=8$

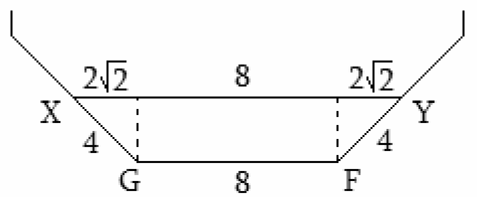
C.

$HI=AB=DA-DB$

$HI=8-6=2$

D.
The sum of the angles in any pentagon is $180(5-2)=540$

11) A.
Draw imaginary altitudes on trapezoid XYFG stemming from F and G. These have a length of $2\sqrt{2}$, since it creates a 45-45-90 triangle with the side of the octagon. This makes the length of $\overline{XY}=8+4\sqrt{2}$



B.
This length is equal to the apothem of the octagon, which is

$$d=\frac{1}{2}HE$$

$$\frac{FG+HE}{2}=XY$$

$$\frac{8+HE}{2}=8+4\sqrt{2}$$

$$8+HE+8\sqrt{2}$$

$$HE=8+8\sqrt{2}$$

$$d=\frac{1}{2}HE=4+4\sqrt{2}$$

C.
This length is equal to the length of the altitude of trapezoid XYFG = $2\sqrt{2}$

D.

$$\frac{Perimeter_{GXZYF}}{Perimeter_{ABCDEFGH}}=\frac{4+8+4\sqrt{2}+8+4\sqrt{2}+4+8}{64}$$

$$\frac{Perimeter_{GXZYF}}{Perimeter_{ABCDEFGH}}=\frac{32+8\sqrt{2}}{64}$$

$$\frac{Perimeter_{GXZYF}}{Perimeter_{ABCDEFGH}}=\frac{4+\sqrt{2}}{8}$$

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12) Josef the Jaguar --

$$D = \frac{12(12-3)}{2}$$

$$D = 54$$

Pamela the Penguin

$$Ext = \frac{360}{36} = 10^\circ$$

$$5 \cdot Ext = 50$$

Alexis the Armadillo

$$s = \frac{2 \cdot Alt}{\sqrt{3}} = \frac{2 \cdot 10\sqrt{3}}{\sqrt{3}} = 20$$

$$P = 3 \cdot 20 = 60$$

Gina the Giraffe

$$A = 360 - (113 + 102 + 88)$$

$$A = 360 - 303 = 57$$

Use trial and error to figure out the measure of the angle. Since one of the guesses was exactly right, there are only four possible correct answers. Two of the guesses were exactly three degrees off, so they are either the same measure, or six degrees apart with the correct guess three degrees away from each. There are no guesses that are exactly the same, so they must be six degrees apart. These two angles are 54 and 60, which means 57 was exactly correct. This holds true since the remaining angle measure, 50, is seven degrees off.

A) Gina the Giraffe
B) Alexis the Armadillo and Josef the Jaguar
C) Pamela the Penguin
D) 57

13) A.
TRUE – since they are coplanar lines. If they weren't coplanar, they could be skew.
B.
TRUE – vertical angles are always equivalent
C.
FALSE – this is true of an indirect proof.

D.
FALSE – sides of ratio a:b have perimeters of length a:b

14) A.
Squares and Rhombi must have perpendicular diagonals. 2.
B.
All parallelograms have bisecting diagonals, so this means that parallelograms, squares, rectangles, and rhombi all have them. 4.

C.
All but general quadrilaterals (i.e. kite) must have two parallel sides. So, trapezoids, parallelograms, squares, rectangles, and rhombi all have at least two. 5.
D.
Only general quadrilaterals and trapezoids can have all differing side lengths. 2.

15) A.
With sides of 4, 6, and 8, the triangle is scalene.
B.

$$4^2 + 6^2 \stackrel{?}{=} 8^2$$

$$52 < 64$$

obtuse

C.
With sides of 16, 16, and 18, the triangle is isosceles.
D.

$$16^2 + 16^2 \stackrel{?}{=} 18^2$$

$$512 > 324$$

acute