1) D -- Pythagorean theorem!

$x^2 = 3^2 + 4^2$

$x = 5$

2) B – Similar Triangles

$\frac{6}{13.5} = \frac{10}{13.5 + x}$

$81 + 6x = 135$

$6x = 54$

$x = 9$

$Shadow = 22.5$

3) A – The two base angles, when added together, will be equal to the measure of the exterior angle of the vertex angle. So, the measure of one base angle is equal to one half of the measure of the exterior angle.

4) A – A line defines the set of points equidistant from two points.

5) C – HH and AA do not prove right triangle congruency. (HH can't even be real)

6) B – The remaining exterior angle is 66 degrees, and the sum of the exterior angles in any heptagon is 360 (as it is with any polygon). So,

$360 = x + 2x + 3x + 4x + 5x + 6x + 66$

$360 = 21x + 66$

$294 = 21x$

$14 = x$

The smallest interior angle will be adjacent to the largest exterior angle, which has an angle measure of 6(14)=84. So, the smallest interior angle has a length of 180-84=96.

7) C – The number of diagonals in a polygon is $\frac{n(n-3)}{2}$. So,

$\frac{n(n-3)}{2} = n^2 - 4n - 7$

$n^2 - 3n = 2n^2 - 8n - 14$

$0 = n^2 - 5n - 14$

$(n-7)(n+2) = 0$

$n = 7$

8) C – Corresponding Parts of Congruent Triangles are Congruent

9) B – Given that $WX = 5$, $VX = 4$, and $VW = 3$, $\triangle VWX$ is a right triangle! So,

$$(WY)^2 = 3^2 + 3^2$$
$$WY = 3\sqrt{2}$$

10) A – In the statement "*The inverse of the contrapositive of the inverse of the converse of the contrapositive of the inverse*," you have 3 inverses, which cancel each other to 1 inverse. You also have 2 contrapositives, which cancel each other out to nothing, as well as 1 converse. 1 inverse and 1 converse become 1 contrapositive. So, $\sim p \to q$ becomes $\sim q \to \sim\sim p$ or $\sim q \to p$.

11) B – The diagonals drawn will be the segments AF, FK, KB, BG, GL, LC, CH, HM, MD, DI, IN, NE, EJ, JA, and then it begins again with segment AF. This method creates 14 diagonals. The total number of diagonals is $\frac{14(11)}{2} = 77$. So the fraction drawn is $\frac{14}{77} = \frac{2}{11}$.

12) D --

$$\frac{LK}{LM} = \frac{JK}{MN}$$
$$\frac{3}{2} = \frac{JK}{3}$$
$$JK = \frac{9}{2} = 4.5$$

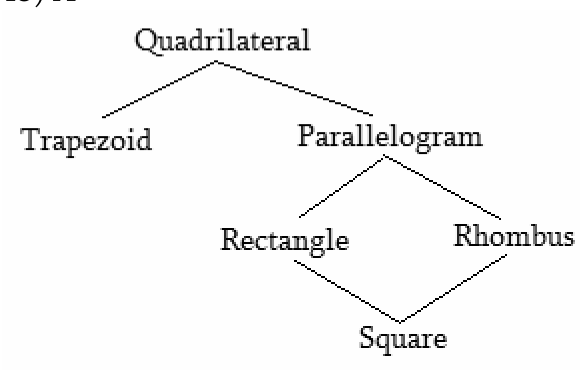
13) B --

$$\sin H = \frac{UG}{GH}$$
$$\cos H = \frac{HU}{GH}$$
$$\sin^2 H + \cos^2 H = \frac{(UG)^2 + (HU)^2}{(GH)^2} = \frac{(GH)^2}{(GH)^2} = 1$$

This is actually true for all values of H!

14) E --
For the first condition, the possible integral values of x fall under $8-6<x<8+6$ or $2<x<14$. These values are 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, and 13.
For the second condition, the possible integral values of x fall under
$64-36<x^2<64+36$ or $28<x^2<100$. These values are 6, 7, 8, and 9. So, there are 4 values.

15) A --



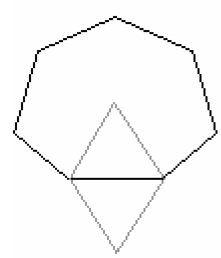
I. True
II. False
III. True (would be a square)
IV. True

16) D --

$\angle FAB = 360-90-51-40=179$

17) A --
Since you are given two sides and the included angle, SAS would be appropriate here.

18) D --
There are going to be two on each diagonal, as well as two on each side. An example of two are shown to the right. So, there are 42 triangles.



19) D --
$$\sum_{n=3}^{6}\left(\frac{360}{n}\right)=\frac{360}{3}+\frac{360}{4}+\frac{360}{5}+\frac{360}{6}=120+90+72+60=342$$

20) B –

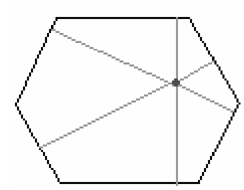
$$x = \frac{ab}{a+b} = \frac{10 \cdot 5}{10+5} = \frac{50}{15} = \frac{10}{3}$$

21) C --
The first important geometer mentioned in history is Thales of Miletus, a Greek who lived about 600 BC. Thales is credited with several simple but important theorems, including the proof that an angle inscribed in a semicircle is a right angle.

22) B --
$\overrightarrow{AI}$ is the angle bisector of $\angle CAR$, so $\frac{AC}{AR} = \frac{IC}{IR}$. So,

$$\frac{4}{8} = \frac{x}{5}$$
$$8x = 20$$
$$2x = 5$$
$$x = \frac{5}{2}$$

23) D --
Given any point in the interior of a hexagon and drawing perpendiculars to all sides, if you take the sum of these lengths, you get the value of six times the length of the apothem, since the lengths pair up to become collinear. A picture is shown to the right. So, the sum of the lengths is 126, which means the apothem has a length of 21. This means that $\frac{1}{2}s = \frac{21}{\sqrt{3}} = 7\sqrt{3}$ so $s = 14\sqrt{3}$.



24) B --
Since the triangle must be acute, the following conditions must hold true.

$$56 + x > 90 \rightarrow x > 34$$
$$56 + x < 180 \rightarrow x < 124$$
$$x < 90$$

So, the value of x must be between 34 and 90, exclusive. This leaves 55 possible values for x. However, the triangle must also be scalene, so the following conditions must also hold true.

$$56 \neq x$$
$$56 \neq 180 - 56 - x \rightarrow x \neq 68$$
$$x \neq 180 - 56 - x \rightarrow x \neq 62$$

So, this eliminates 3 possible values for x, leaving 52 possible values.
25) C --

The sum of the squares of the medians is

$$\left(\sqrt{\frac{a^2}{2}+\frac{b^2}{2}-\frac{c^2}{4}}\right)^2+\left(\sqrt{\frac{a^2}{2}+\frac{c^2}{2}-\frac{b^2}{4}}\right)^2+\left(\sqrt{\frac{c^2}{2}+\frac{b^2}{2}-\frac{a^2}{4}}\right)^2$$

, which looks daunting, until it simplifies…

$$\frac{a^2}{2}+\frac{b^2}{2}-\frac{c^2}{4}+\frac{a^2}{2}+\frac{c^2}{2}-\frac{b^2}{4}+\frac{c^2}{2}+\frac{b^2}{2}-\frac{a^2}{4}$$

$$\frac{2a^2}{4}+\frac{2b^2}{4}-\frac{c^2}{4}+\frac{2a^2}{4}+\frac{2c^2}{4}-\frac{b^2}{4}+\frac{2c^2}{4}+\frac{2b^2}{4}-\frac{a^2}{4}$$

$$\frac{3a^2}{4}+\frac{3b^2}{4}+\frac{3c^2}{4}$$

$$\frac{3}{4}\left(a^2+b^2+c^2\right)$$

So, the sum of the squares of the medians of the triangle is ¾ the sum of the square of the sides. So, the ratio is 4:3.

26) E --

I. TRUE. The supplement of the complement of the supplement will be equal to the supplement of the angle. However, since the complement can be taken, the supplement of the original angle MUST be acute. So the original angle MUST be obtuse.

II. FALSE. $180-\left(90-\left(180-95\right)\right)=175$.

III. FALSE.

$$\angle GUM = x$$

$$\angle UGM = \frac{180-\angle GUM}{2}$$

$$180-\left(90-\left(180-\angle GUM\right)\right)=270-\angle GUM$$

$$\frac{1}{2}\left(180-\angle UGM\right)=\frac{1}{2}\left(180-\frac{180-\angle GUM}{2}\right)$$

$$\frac{1}{2}\left(180-\angle UGM\right)=\frac{180+\angle GUM}{4}\neq 270-\angle GUM$$

27) D --

Skew lines never intersect!

28) C --

Planes cannot intersect at a point, since they go on forever. Most planes intersect at a line, and planes that coincide intersect at a plane.

29) A --

$3^2 + 4^2 = 5^2$

$(3n)^2 + (4n)^2 = (5n)^2$

$3^{2n} + 4^{2n} \neq 5^{2n}$

$(3+n)^2 + (4+n)^2 \neq (5+n)^2$

$(3-n)^2 + (4-n)^2 \neq (5-n)^2$

$n^6 + n^8 \neq n^{10}$

30) A --

$\angle YZU \cong \angle WZV$, since they are vertical angles. Since $\overline{TXY} \parallel \overline{WZU}$, $\angle ZYX \cong \angle WZV$. Since $m\angle UZV = 115$, $m\angle YZU = 115$. So, $m\angle ZYX + m\angle WZV = 65 + 65 = 130$.