

$$1. \frac{1}{2} \int_0^{3/2} \frac{2dx}{\sqrt{9-4x^2}} = \frac{1}{2} \arcsin \frac{2(3/2)}{3} - \frac{1}{2} \arcsin \frac{2(0)}{3} = \left(\frac{1}{2} \arcsin 1 - \frac{1}{2} \arcsin 0\right) = \left(\frac{1}{2}\right)\left(\frac{\pi}{2}\right) = \frac{\pi}{4} \Rightarrow \mathbf{B}$$

$$2. y = \frac{x}{\sqrt{x^2+7}} \Rightarrow y^2 = \frac{x^2}{x^2+7} \Rightarrow x^2 y^2 + 7y^2 = x^2 \Rightarrow \frac{7y^2}{1-y^2} = x^2 \Rightarrow \frac{y\sqrt{7}}{\sqrt{1-y^2}} = x \dots$$

switch the x's and y's so that $f^{-1}(x) = \frac{x\sqrt{7}}{\sqrt{1-x^2}} \Rightarrow A = 7$ and $B = 1 \Rightarrow A + B = 8$

$\Rightarrow \mathbf{C}$

$$3. \text{ Multiply both top and bottom by } \frac{1}{\sqrt{x^2}} \text{ and the limit } = \frac{-68}{\sqrt{6}} = \frac{-34\sqrt{6}}{3} \Rightarrow \mathbf{E}$$

$$4. 6(x^2 + y^2)(2x + 2yy') = 100(xy' + y) \dots \text{now plug in } (3,1) \text{ and solve...}$$

$$6(9 + 1)(6 + 2y') = 100(3y' + 1) \Rightarrow 360 + 120y' = 300y' + 100 \Rightarrow 180y' = 260 \Rightarrow y' = 26/18 = 13/9 \Rightarrow \text{slope of normal} = -9/13 \text{ and the equation of the line through } (3,1) \text{ with slope } -9/13 \text{ is } 9x + 13y = 40 \Rightarrow \mathbf{A}$$

$$5. \frac{dy}{dx} = \frac{(4 \cos^3 x)(-\sin x)}{(\ln 8)(\cos^4 x)} = \frac{-4 \sin x}{(3 \ln 2)(\cos x)} = \frac{-4 \tan x}{3 \ln 2} \Rightarrow \mathbf{D}$$

$$6. 2x + 2y = M \Rightarrow A = xy = x\left(\frac{M-2x}{2}\right) = \frac{Mx-2x^2}{2} \Rightarrow \frac{dA}{dx} = \frac{M}{2} - 2x \dots \text{to find}$$

critical points we set equal to 0 $\Rightarrow 0 = \frac{M}{2} - 2x \Rightarrow x = \frac{M}{4}$ and plugging into first

equation above we get $y = \frac{M}{4} \Rightarrow \text{maximum area} = \left(\frac{M}{4}\right)\left(\frac{M}{4}\right) = \frac{M^2}{16} \Rightarrow \mathbf{A}$

$$7. \lim_{x \rightarrow 0} \frac{16 \sin(8x) \cos(8x)}{x} = \lim_{x \rightarrow 0} \frac{16 \cos(8x)}{1} \cdot \frac{8 \sin(8x)}{8x} = \lim_{x \rightarrow 0} \frac{16 \cos(8x)}{1} (8)(1) = \lim_{x \rightarrow 0} (16 \cos(8x))(8) = 16(8)(\cos 0) = 128 \Rightarrow \mathbf{E}$$

$$8. 12.5 \int_0^{\pi} \sin \theta d\theta = (12.5)(2) = 25 \Rightarrow \mathbf{A}$$

$$9. h' = fg' + gf' = (|x^2 - 4|)(-|x| \sin x + \frac{x}{|x|} \cos x) + (|x| \cos x) \left(\frac{2x^3 - 8x}{|x^2 - 4|}\right) \dots$$

note: $\frac{d(|u|)}{dx} = \frac{u'u}{|u|} \dots \text{plugging in } x = 1 \Rightarrow 3(-\sin 1 + \cos 1) + (\cos 1)(-2) =$

$\cos 1 - 3\sin 1 \Rightarrow \mathbf{C}$

$$10. \text{ Area} = \int_a^b (f(x) - g(x)) dx, \text{ where } a \text{ and } b \text{ are the } x \text{ coordinates of the intersection}$$

points of the curves, and $f(x)$ is the top curve, and $g(x)$ is the bottom curve. There are three intersection points and for $[-1,0]$ $f(x)$ is the top curve, but from $[0,1]$ $g(x)$ is the top curve, so to find the area you evaluate

$$\text{Area} = \int_{-1}^0 (4x^3 - 4x) dx + \int_0^1 (4x - 4x^3) dx = x^4 - 2x^2 \Big|_{-1}^0 + 2x^2 - x^4 \Big|_0^1 = 1 + 1 = 2. \Rightarrow \mathbf{C}$$

$$11. y' = -3 \sin 12t - 4 \cos 12t \Rightarrow \text{at } t = \frac{\pi}{8} \text{ we get } -3 \sin \frac{3\pi}{2} - 4 \cos \frac{3\pi}{2} = -3(-1) = 3$$

$\Rightarrow \mathbf{A}$

$$12. y' = (2x \cos 2x - \sin 2x) \left(e^{\frac{\sin 2x}{x}} \right) \Rightarrow y'(1) = (2 \cos 2 - \sin 2)(e^{\sin 2}) \Rightarrow \mathbf{D}$$

$$13. V = \frac{\pi r^2 h}{3} \text{ and } \frac{7}{10} = \frac{r}{h} \Rightarrow r = 7h/10 \Rightarrow V = \frac{49\pi h^3}{300} \Rightarrow \frac{dv}{dt} = \left(\frac{49\pi}{100}\right)(h^2) \left(\frac{dh}{dt}\right) \Rightarrow$$

$$8 = \left(\frac{49\pi}{100}\right)(36) \left(\frac{dh}{dt}\right) \Rightarrow \frac{2}{9} = \left(\frac{49\pi}{100}\right) \left(\frac{dh}{dt}\right) \Rightarrow \frac{dh}{dt} = \frac{200}{441\pi} \Rightarrow \mathbf{B}$$

14. Looking for critical points where $f(x)$ is defined...on $[8, \infty]$ we find none inside of the interval so, using the endpoint $x = 8$ we find that the point $(8, 0)$ is the closest point on the graph to $(2,0)$...or simply draw the graph \Rightarrow sum = 8 $\Rightarrow \mathbf{A}$

$$15. \frac{\pi}{8} (\cos^2 0 + 2 \cos^2 \frac{\pi}{4} + 2 \cos^2 \frac{\pi}{2} + 2 \cos^2 \frac{3\pi}{4} + \cos^2 \pi) = \frac{\pi}{8} (1 + 1 + 0 + 1 + 1) = \frac{\pi}{2} \Rightarrow \mathbf{D}$$

$$16. \text{L'hopital's rule } \Rightarrow \lim = \lim_{x \rightarrow 1} \frac{\frac{6}{1+x^2}}{\frac{6}{1}} = \frac{2}{1} = 3 \Rightarrow \mathbf{A}$$

17. By definition $e = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$. This can be extended to show that

$$e^{a/b} = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{bx}\right)^{ax}. \text{ So we get } e^{2/3} \Rightarrow \mathbf{C}$$

$$18. 1393/398 = 3.5 \Rightarrow 28 \left(\frac{1}{2}\right)^{3.5} = 28 \left(\frac{1}{8}\right) \left(\frac{\sqrt{2}}{2}\right) = \frac{7\sqrt{2}}{4} \Rightarrow \mathbf{D}$$

19. Taking the natural log of both sides we get $\ln y = \ln \frac{(x-2)^2}{\sqrt{x^2+1}} \Rightarrow$

$$\ln y = 2 \ln(x-2) - \frac{1}{2} \ln(x^2+1) \Rightarrow \frac{y'}{y} = \frac{2}{x-2} - \frac{x}{x^2+1} \Rightarrow$$

$$y' = \frac{(x-2)^2}{\sqrt{x^2+1}} \left[\frac{2}{x-2} - \frac{x}{x^2+1} \right] \Rightarrow y'(0) = \left(\frac{4}{1}\right) \left(\frac{2}{-2}\right) = 4(-1) = -4 \Rightarrow \mathbf{A}$$

20. To find the minimum value of the second derivative, we must set the 3rd derivative equal to zero... $y' = 4x^3 + 18x^2 + 4$, $y'' = 12x^2 + 36x$, $y''' = 24x + 36$
 $\Rightarrow 0 = 24x + 36 \Rightarrow 24x = -36 \Rightarrow x = -3/2$ (upon testing we find this to be a minimum) $\Rightarrow y''(-3/2) = 12(9/4) + 36(-3/2) = 27 - 54 = -27 \Rightarrow \mathbf{C}$

21. $f'(x) = \frac{10}{3}x^{1/3}(x^{1/3} - 2)$ and $f''(x) = \frac{20(x^{1/3} - 1)}{9x^{2/3}}$. The function has two critical

numbers at $x = 0$ and $x = 8$ and two possible points of inflection at $x = 0$ and $x = 1$. The domain is all real numbers. $f'(x)$ is positive and $f''(x)$ is negative on $-\infty < x < 0$, so (A) is true. Continuing the analysis in the same way, we find (B) and (C) to be true. Since $f''(x)$ is positive on $8 < x < \infty$, the graph is concave upward on that interval and (D) is false \Rightarrow **D**

22. Sum of squares from 1 to n is given by $\frac{n(n+1)(2n+1)}{6}$...plugging in $n = 16$

gives $\frac{16(17)(33)}{6} = (8)(17)(11) = (88)(17) = 1496 \Rightarrow$ **B**

23. $\frac{1}{-1 - (-3)} \int_{-3}^{-1} (x^2 + 6x + 10) dx = \frac{1}{2} \left[\frac{x^3}{3} + 3x^2 + 10x \right]$ evaluated from -3 to $-1 =$

$\frac{1}{2} \left[\left(\frac{-1}{3} + 3 - 10 \right) - \left(-9 + 27 - 30 \right) \right] = \frac{1}{2} \left(\frac{-22}{3} + 12 \right) = \frac{1}{2} \left(\frac{14}{3} \right) = \frac{7}{3} \Rightarrow$ **A**

24. $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ and $f'(x) = 6x$. $x_2 = 1 - \frac{2}{6} = \frac{2}{3} \Rightarrow$

$x_3 = \frac{2}{3} - \frac{1/3}{4} = \frac{2}{3} - \frac{1}{12} = \frac{7}{12} \Rightarrow$ **A**

25. $\frac{(1 - \cos \theta)(\cos \theta) - (\sin \theta)(\sin \theta)}{(1 - \cos \theta)^2} = \frac{\cos \theta - \cos^2 \theta - \sin^2 \theta}{(1 - \cos \theta)^2} = \frac{\cos \theta - 1}{(1 - \cos \theta)^2} = \frac{-1}{1 - \cos \theta}$

\Rightarrow **E**

26. $P = \frac{Tk}{V^2} \Rightarrow P' = \frac{-2Tk}{V^3} \Rightarrow$ **C**

27. $= 6 \int_0^{\pi/6} \sec x dx$ (note: we can eliminate the absolute value when removing from the radical because it's positive from 0 to $\pi/6$) $= 6 \ln |\sec x + \tan x|$ evaluated from 0

to $\pi/6 = 6 \left(\ln \left| \frac{2\sqrt{3}}{3} + \frac{\sqrt{3}}{3} \right| - \ln |1| \right) = 6 \ln \sqrt{3} = 3 \ln 3 \Rightarrow$ **A**

28. The parrot population is growing fastest when the 1st derivative is at a

maximum. So $P'(t) = \frac{200t(t^2 + 1) - 100t^2(2t)}{(t^2 + 1)^2} = \frac{200t}{(t^2 + 1)^2}$, then

$$P''(t) = \frac{200(t^2 + 1)^2 - 200t[2(t^2 + 1)(2t)]}{(t^2 + 1)^4} = \frac{200(t^2 + 1)[(t^2 + 1) - 2(2t)(t)]}{(t^2 + 1)^4} = \frac{200(-3t^2 + 1)}{(t^2 + 1)^3}$$

. $P''(t)$ has a critical value when the numerator is 0 (the denominator cannot be 0 since setting it equal to 0 gives imaginary roots). So the

critical values we get by setting the numerator equal to 0 are $t = \pm\sqrt{\frac{1}{3}}$. By

the domain restrictions, the only possible t is $t = \sqrt{\frac{1}{3}} = \frac{\sqrt{3}}{3}$. Testing this

value shows that this is in fact the value that maximizes P'' . \Rightarrow C

$$29. \frac{dy}{d(x^2 - 16x)} = \frac{dy/dx}{d(x^2 - 16x)/dx} = \frac{6x^2 - 30x - 144}{2x - 16} =$$

$$\frac{3x^2 - 15x - 72}{x - 8} = \frac{(x - 8)(3x + 9)}{x - 8} = 3x + 9 \Rightarrow \mathbf{C}$$

30. $A(x - 5) + B(x + 6) = x - 27$. Plugging in $x = 5$ to eliminate A , we get $11B = -22$
 $\Rightarrow B = -2 \Rightarrow \mathbf{B}$

1. B
2. C
3. E
4. A

- 5. D
- 6. A
- 7. E
- 8. A
- 9. C
- 10.C
- 11.A
- 12.D
- 13.B
- 14.A
- 15.D
- 16.A
- 17.C
- 18.D
- 19.A
- 20.C
- 21.D
- 22.B
- 23.A
- 24.A
- 25.E
- 26.C
- 27.A
- 28.C
- 29.C
- 30.B