Algebra II Individual Solutions

1. Since the day is made up entire of A and B we can determine that $A+B=24$. Now also $|A+Bi|=\sqrt{A^2+B^2}=(\sqrt{386})^2 \Rightarrow A^2+B^2=386$. Now notice by substitution that $A^2+(24-A)^2=386 \Rightarrow 2A^2-48A+576=386 \Rightarrow A^2-24A+95=0 \Rightarrow A=5,19$. A can be either 5 or 19, but substituting these values back into the original equation gives the values of B as 5 or 19, so we just need to know that one is 5 while one is 19. Thus the positive difference is 14 which is closest to 18.

2.
$$\frac{97-(4\div 6+4)\cdot 9+2y-5+\left(6^{7+5(-1)^3}\right)y}{y+8-3(4+5\cdot 7)+60-3}=1$$
$$\frac{97-42+2y-5+36y}{y+8-117+60-3}=1$$
$$50+38y=y-52$$
$$37y=-102$$
$$y=\frac{-102}{37}$$

3. The range of $\sqrt{-x+1}$ is $[0,\infty)$. The range of $-\sqrt{-x+1}$ is $(-\infty,0]$. So the range of $2-\sqrt{-x+1}$ is $(-\infty,2]$.

4. Completing the square yields $(x^2-10x+25)+(y^2+6y+9)=-34+25+9 \Rightarrow (x-5)^2+(y+3)^2=0$. This is the equation of a point, thus E.

5. The midpoint of the segment is $(\frac{2+7}{2},\frac{-13+0}{2})\equiv(\frac{9}{2},\frac{-13}{2})$. The slope of the segment is $\frac{-13-0}{2-7}=\frac{-13}{-5}=\frac{13}{5}$. So the slope of the line perpendicular is $\frac{-5}{13}$. Thus, the line is $(y-(\frac{-13}{2}))=\frac{-5}{13}(x-\frac{9}{2}) \Rightarrow 13y+\frac{169}{2}=-5x+\frac{45}{2} \Rightarrow 5x+13y=-62$

6.

$$g(x) = 4\ln x + 1$$
$$x = 4\ln g^{-1}(x) + 1$$
$$\frac{x-1}{4} = \ln g^{-1}(x)$$
$$g^{-1}(x) = e^{\frac{x-1}{4}}$$
$$g^{-1}(\ln 81 + 1) = e^{\frac{\ln 81 + 1 - 1}{4}}$$
$$g^{-1}(\ln 81 + 1) = e^{\frac{\ln 81}{4}}$$
$$g^{-1}(\ln 81 + 1) = e^{\ln 3} = 3$$

7.

$$11111011000_2 = [1(2^{10}) + 1(2^9) + 1(2^8) + 1(2^7) + 1(2^6) + 0(2^5) + 1(2^4) + 1(2^3) + 0 + 0 + 0]_{10}$$
$$2008_{10}$$

Summing the digits of the results gives 10 and squaring this is 100 which is closest to 68.

8. The line has positive slope, with x-intercept (2,0) and y-intercept (0,-7). Drawing this out shows that the line passes through only quadrants I, III and IV.

9. This inequality is equivalent to $|x-3| \geq 4$. This means $x - 3 \geq 4 \Rightarrow x \geq 7$ or $x - 3 \leq -4 \Rightarrow x \leq -1$. So the integral solutions that work are (7,8,9,10,…) or (-1,-2,-3,-4,…). Now looking at the sum

$$(-1 + -2 + -3 + -4 + -5 + -6 + -7 + -8 + -9 + -10 + \ldots) + (7 + 8 + 9 + 10) \Rightarrow$$
$$(-1 + -2 + -3 + -4 + -5 + -6) + (-7 + 7) + (-8 + 8) + (-9 + 9) + \ldots \Rightarrow$$
$$(-21) + 0 + 0 + 0 + \ldots = -21$$

10. $\frac{6x}{(x+4)^2} = \frac{A(x+4)+B}{(x+4)^2}$. So 6x=Ax+4A+B. So A=6. Then 4(6)+B=0. So B=-24. So 6-(-24) = 30

11. Leonhard Euler

12. To find the remainder we can substitute $x+1=0 \Rightarrow x=-1$ into the numerator of the function. So the remainder is
$(-1)^{100}-(-1)^{99}+(-1)^{98}-(-1)^{97}+\ldots+(-1)^{2}-(-1)+1=1+1+1+\ldots+1+1+1=101$

13. Let the digits (in ascending order) by A, B, C, D. Using logic notice that C is the only digit that can be divisible by 2 which means C can be 2,4,6 or 8. Using logic, C cannot be 2, since A and B are both less than C. Trying C=4, means that B=3 and A=1, so D must be 7 since D=B+C. Now,
$\frac{A+B+C+D}{A+B}=3.75 \Rightarrow \frac{1+3+4+7}{1+3}=3.75$. So these are the values, and the product is 1(3)(4)(7) = 84 which is closest to 80.

14. The consecutive integers are n, n+1 and n+2. So
$n^2+(n+1)^2+(n+2)^2-(n+n+1+n+2)=20 \Rightarrow 3n^2+3n-18=0 \Rightarrow n=-3,2$. By the stipulation that most numbers are odd, n=-3 which means the 3 integers are -3, -2 and -1. So the product is -6 which is closest to -4.

15. The slopes of any hyperbola are exact opposites of one another. Thus the sum of the slopes is 0.

16. The product of the roots can be written as $\frac{C}{A}$ and the sum of the roots can be written as $\frac{-B}{A}$. So, $\frac{C}{A}=2(\frac{-B}{A})$ Now, the discriminant is
$B^2-4AC=49 \Rightarrow 1-\frac{4AC}{B^2}=\frac{49}{B^2} \Rightarrow 1-4(\frac{A}{B})(\frac{C}{B})=\frac{49}{B^2} \Rightarrow 1-4(\frac{A}{B})(\frac{C}{A})(\frac{A}{B})=\frac{49}{B^2}$
$\Rightarrow \frac{B^2-49}{B^2}=-4(\frac{A}{B})^2(\frac{C}{A}) \Rightarrow \frac{B^2-49}{4B^2}=-(\frac{A}{B})^2(\frac{-2B}{A}) \Rightarrow \frac{B^2-49}{4B^2}=\frac{2A}{B} \Rightarrow A=\frac{B^2-49}{8B}$

17.

$$\frac{xy^2z + x^{-2}y}{y^{-1}z^3}$$

$$\frac{xy^2z + \frac{y}{x^2}}{\frac{z^3}{y}}$$

$$\frac{x^3y^2z + y}{x^2} \cdot \frac{y}{z^3}$$

$$\frac{x^3y^3z + y^2}{x^2z^3}$$

$$\frac{y^2\left(x^3yz + 1\right)}{x^2z^3}$$

18. Det(M) = $x(2x-4)-2(2x+4)=2x^2-8x-8$. Now the smallest prime is 2. So det(M)=2. So $2x^2-8x-8=2 \Rightarrow 2x^2-8x-10=0 \Rightarrow x^2-4x-5=0 \Rightarrow x=-1{,}5$. So none of the given intervals contain both answers, thus E.

19. $\sqrt{x+\sqrt{x+...}}=n \Rightarrow \sqrt{x+n}=n$. Now also $4n=28-x \Rightarrow x=28-4n$. By substitution $\sqrt{28-4n+n}=n \Rightarrow 28-3n=n^2 \Rightarrow n^2+3n-28=0 \Rightarrow n=-7{,}4$. Now n must be 4 since it is the sum of square roots. From this information, and substitution we find that x = 12. So $\frac{n}{x}=\frac{4}{12}=\frac{1}{3}$ which is closest to $\frac{1}{4}$.

20.

$$x^2+y^2+Cx+Dy=E$$
$$3C-D+E=10$$
$$6C+4D-E=-52$$
$$4C+8D-E=-80$$
$$9C+3D=-42 \to 3C+D=-14$$
$$7C+7D=-70 \to C+D=-10$$
$$C=-2$$
$$D=-8$$
$$E=8$$
$$x^2-2x+y^2-8y=8$$
$$x^2-2x+1+y^2-8y+16=8+1+16$$
$$(x-1)^2+(y-4)^2=25$$

21. $(f \circ g)(x)=\dfrac{\dfrac{1}{\sqrt{x}-1}+3}{(\dfrac{1}{\sqrt{x}-1})^2-1}$. There are two problems that must be looked at

$\sqrt{x}-1 \neq 0 \Rightarrow x \neq 1$. Also, $(\dfrac{1}{\sqrt{x}-1})^2-1 \neq 0 \Rightarrow \dfrac{1}{\sqrt{x}-1} \neq \pm 1$. Solving for x, yields that $x \neq 0,4$. So the product is (0)(1)(4)=0

22.

$$o+t=7 \Rightarrow t=7-o$$
$$10o+15t+0.05(10o+15t)=94.50 \Rightarrow 10o+15(7-o)+0.05(10o+15(7-o))=94.50$$
$$\Rightarrow o=3 \Rightarrow t=7-3=4$$

Now, since the friends eat half of the two-topping pizzas, 2 two-toppings are left and 3 one-toppings. They then eat half of the remaining pizzas. So there are 1.5 one topping and 1 two topping for a total of 20 slices left over. So he makes a sale of $20.

23. Multiplying we get $\dfrac{3^{2009}+3^{2008}-3^{2006}}{3^{2005}+3^{2004}}$. Dividing numerator and denominator by 3^{2004} we get $\dfrac{3^5+3^4-3^2}{3+1}=78.75$ which is closest to 100.

24.

$$5 \oplus 3 = (3-1)\frac{5 \oplus 1}{2} = \ln 2$$

$$5 \oplus 5 = (5-1)\frac{5 \oplus 3}{2} = 2\ln 2$$

$$5 \oplus 7 = (7-1)\frac{5 \oplus 5}{2} = 6\ln 2 = \ln 64$$

25. A is false because the graph could increase down past the y-intercept. B is true since there are no real solutions. C is false because the sum of the roots will be the sum of the real parts of the complex solutions and if b=0 that implies the sum of the roots is 0.

26. First, notice that $\frac{1}{3}\log(25) = A \Rightarrow 2\log(5) = 3A \Rightarrow \log(5) = \frac{3A}{2}$, and $-2\log(7) = B \Rightarrow \log(7) = \frac{-B}{2}$. Now,

$$\log(5\sqrt{35}) = \log(5\sqrt{5}\sqrt{7}) = \log(5^{3/2}) + \log(7^{1/2}) = \frac{3}{2}\log(5) + \frac{1}{2}\log(7) = \frac{3}{2}(\frac{3A}{2}) + \frac{1}{2}(\frac{-B}{2})$$

$$= \frac{9A - B}{4}$$

27. Rewrite as $2\sqrt{x^3} - 6x - 20\sqrt{x} = 0$. Making the substitution $u = \sqrt{x}$ yields, $2u^3 - 6u^2 - 20u = 0$. Solving for u yields:
$u^3 - 3u^2 - 10u = 0 \Rightarrow u(u^2 - 3u - 10) = 0 \Rightarrow u(u-5)(u+2) = 0 \Rightarrow u = 0,5,-2$.
Substituting these values back into the initial substitution we get that x=0 and 25. U= -2 does not yield a value of x. Therefore the sum of the solutions is 25.

28. First, notice that the length of the semi-minor axis is $\frac{4-(-8)}{2} = 6$. Now, let c be the distance from the center of the ellipse to one of the foci, then $10^2 = 6^2 + c^2 \Rightarrow c = 8$. So the distance between the 2 foci is 2(8)=16 which is closest to 15.

29. First, $\sqrt{f(-i)*5+3}=\sqrt{2*5+3}=\sqrt{13}$. Next,

$|\frac{f(1)}{g(1)}|=|\frac{1-i}{2-3i}\bullet\frac{2+3i}{2+3i}|=|\frac{5+i}{13}|=\sqrt{(\frac{5}{13})^2+(\frac{1}{13})^2}=\frac{\sqrt{2}}{\sqrt{13}}$. Finally, the product is $\sqrt{13}\bullet\frac{\sqrt{2}}{\sqrt{13}}=\sqrt{2}$. Which is closest to $2\sqrt{2}$.

30. Notice that $\sqrt[3]{1000}<\sqrt[3]{2008}<\sqrt[3]{1000000}\Rightarrow 10<\sqrt[3]{2008}<100$, so the value has 2 digits to the left of the decimal.