Team Solutions

1. **A**. $R = kL \Rightarrow k = \frac{R}{L}$

   B. $x = kyz \Rightarrow (-7) = k(210)(-10) \Rightarrow k = \frac{1}{300}$

   $-2 = (\frac{1}{300})(y)(25) \Rightarrow y = -24$

   C. $V = k(\frac{T}{P}) \Rightarrow 33 = k(\frac{110}{15}) \Rightarrow k = 4.5$

   $V = 4.5(\frac{180}{28}) = \frac{405}{14}$

   D. Solving the system yields $x = \frac{5}{3}, y = \frac{1}{3}, z = \frac{-5}{3}$. Now, the variations is

   $z = k(\frac{\sqrt{x}}{y^3}) \Rightarrow \frac{-5}{3} = k(\frac{\sqrt{5/3}}{(1/3)^3}) \Rightarrow -45 = k(\frac{\sqrt{15}}{3}) \Rightarrow k = -9\sqrt{15}$

2. **A**. Completing the square yields $(x^2 - 2x + 1) + (y^2 - 2y + 1) = 2 + 1 + 1 \Rightarrow (x-1)^2 + (y-1)^2 = 4$. The shortest distance is the distance from the point to the center of the circle minus the radius so, with the point (9,5) and the center (1,1), we get $d = \sqrt{(9-1)^2 + (5-1)^2} - 2 = 4\sqrt{5} - 2$

   B. Exactly ¼ of the circle falls in the fourth quadrant, so $A = \frac{22\pi}{4} = \frac{11\pi}{2}$

   C. Completing the square yields

   $16(x^2 - 4x + 4) + 9y^2 = 80 + 64 \Rightarrow \frac{(x-2)^2}{9} + \frac{y^2}{16} = 1$. So the length of the semi-minor axis is 3 and the length of the semi-major axis is 4, so the area is $3(4)\pi = 12\pi$

   D. Completing the square yields that the equation is $\frac{(x+3)^2}{1} + \frac{(y-1)^2}{9} = 1$. Notice the center is at (-3,1). Now, the point with the highest x-value is the point directly to the right of the center. This point is (-2,1). The distance from this point to a focus is equal to the length of the half-major-axis, which is 3.

3. **A**. We can have that $2x - 3 = -7x + 5 \Rightarrow 9x = 8 \Rightarrow x = \frac{8}{9}$. Or

   $-2x + 3 = -7x + 5 \Rightarrow 5x = 2 \Rightarrow x = \frac{2}{5}$. So $\frac{8}{9} + \frac{2}{5} = \frac{58}{45}$

B. First, $x^2+7x+14=1 \Rightarrow x^2+7x+13=0$, so the product of the roots is $\frac{13}{1}=13$. Also, $x^2+7x+14=-1 \Rightarrow x^2+7x+15=0$, so the product of the roots is 15. So the total product of all of the roots is 13(15)=195.

C. Solving the first inequality we get $x \geq 2, x \leq \frac{-16}{3}$. Solving the second inequality we get -4<x<-2. Since these 2 do not intersect anywhere, there are 0 integral solutions.

D. Since none of the functions pass the vertical line test, 0 of the functions are one-to-one.

4. **A**. Slope $= \frac{7-3}{-2-3} = \frac{-4}{5}$. So the equation of the line is

$(y-3) = \frac{-4}{5}(x-3) \Rightarrow 4x+5y=27$

B. The slope of the line through the given points is $\frac{-1-(-2)}{-2-5} = \frac{-1}{7}$. So the perpendicular slope is 7. The line through the given points is $(y+1) = \frac{-1}{7}(x+2)$, substituting in y=0, gives the x intercept of (-9,0). So the perpendicular line is $(y-0)=7(x+9) \Rightarrow 7x-y=-63$

C. $\frac{2(0)+4(2)+(-3)}{\sqrt{2^2+4^2}} = \frac{5}{\sqrt{20}} = \frac{\sqrt{5}}{2}$

D. Since the lines are parallel first notice that a point on the first listed line is (0,-5). Now, use the same formula as above with the point and the second line.
$\frac{2(0)+(-1)(-5)+(-3)}{\sqrt{2^2+1^2}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

5. **A**. If the function has exactly one real root, then the discriminant is 0. So $m^2-4(-2)(-7)=0 \Rightarrow m^2=56 \Rightarrow m=\pm\sqrt{56}$. The product of the possible values for m is -56.

B. Since this is an upward opening parabola, the minimum value occurs at the vertex. The x-value of the vertex is $\frac{-(-5)}{2(3)} = \frac{5}{6}$. So the minimum value is

$3(\frac{5}{6})^2 - 5(\frac{5}{6}) + 6 = \frac{47}{12}$

C. Notice that the sum of the roots is $-2 = \frac{-b}{a} \Rightarrow b = 2a$ and the product of the roots is $-24 = \frac{c}{a} \Rightarrow c = -24a$. Now the discriminant is 4, so

$b^2 - 4ac = 4 \Rightarrow (2a)^2 - 4a(-24a) = 4 \Rightarrow 100a^2 = 4 \Rightarrow a = \frac{1}{5}$ since a>0. Using substitution we find that $b = 2(\frac{1}{5}) = \frac{2}{5}$ and $c = -24(\frac{1}{5}) = \frac{-24}{5}$. So the sum is $\frac{-21}{5}$.

D. The rational root theorem says that only possible rational roots for a function can be found by taking the factors of the constant term and dividing by the factors of the coefficient of the quadratic term. The factors of the constant term are $\pm 1, \pm 2, \pm 4$ and the factors of the coefficient of the quadratic term are $\pm 1, \pm 2, \pm 3, \pm 6$. By dividing and taking the unique answers, we get that there are 16 possible rational roots.

6. **A**. $(2i+1)^6 = [(2i+1)^2]^3 = (4i-3)(4i-3)(4i-3) = (-24i-7)(4i-3) = 117i + 44$. So the sum of a and b is 161.
   B. When 214567 is divided by 4 the remainder is 3, so this is equivalent to $i^3 = -i$
   C. The sum of all the roots of the function is 3, so the sum of the other 3 roots is 3-(1+i)=(2-i)
   D. $\frac{6+i}{5-2i} \bullet \frac{5+2i}{5+2i} = \frac{28+17i}{29} = \frac{17}{29}i + \frac{28}{29}$, so the sum of a and b is $\frac{45}{29}$

7. **A.** $\sqrt{-4x + \sqrt{-4x + \sqrt{-4x + ...}}} = x + 2 \Rightarrow \sqrt{-4x + (x+2)} = x + 2 \Rightarrow -3x + 2 = (x+2)^2$
   $x^2 + 7x + 2 = 0 \Rightarrow x = \frac{\sqrt{41} - 7}{2}$. Only this value of x gives a positive value for the square root.
   B. $\frac{7}{6x^2 + 13x + 6} = \frac{A}{2x+3} + \frac{B}{3x+2} \Rightarrow \frac{7}{6x^2 + 13x + 6} = \frac{A(3x+2) + B(2x+3)}{6x^2 + 13x + 6}$.
   So solving we get that 3A+2B=0 and 2A+3B=7, solving this system yields $A = \frac{-14}{5}, B = \frac{21}{5}$. The sum is $\frac{7}{5}$.
   C. $\frac{1}{1 + \frac{1}{x+3}} - \frac{1}{1 + \frac{1}{x}} = \frac{3}{28} \Rightarrow \frac{x+3}{x+4} - \frac{x}{x+1} = \frac{3}{28} \Rightarrow \frac{(x+3)(x+1) - x(x+4)}{(x+4)(x+1)} = \frac{3}{28}$
   $\Rightarrow 28(3) = 3(x^2 + 5x + 4) \Rightarrow 3x^2 + 15x - 72 = 0 \Rightarrow x = 3$ since x>0.

   D. $(\sqrt{x+5} - \sqrt{2x}) = 0 \Rightarrow \sqrt{x+5} = \sqrt{2x} \Rightarrow x + 5 = 2x \Rightarrow x = 5$

8. **A.** $\ln(14400) = \ln(2^6 \cdot 3^2 \cdot 5^2) = 6\ln(2) + 2\ln(3) + 2\ln(5) = 6a + 2b + 2c$
   B. Let $u = 2^x$, then this equation becomes $u^2 + u - 6 = 0 \Rightarrow u = 2, -3$. The only root that makes sense is u=2, so back substituting we get x=1.
   C. After 1 year we have 1+1(.04)=1.04. After the second year we have 1.04+1.04(.04)=1.08 to 2 decimals.

D. $A = 1e^{\frac{\ln(50)}{100}(200)} = e^{2\ln(50)} = e^{\ln(2500)} = 2500$

9. **A.** Completing the square we get that $y-7 = 2(x^2 + \frac{3}{2}x + \frac{9}{16}) \Rightarrow \frac{1}{2}(y-7) = (x-\frac{3}{4})^2$. So the length of the latus rectum is ½.
B. Substituting the values into the function, we get (a-b+c=18), (a+b+c=6) and (4a+2b+c=15). Solving this system of equations yields a=5, b=-6 and c=7. So a+b+c=6.
C. Completing the square we get
$4x^2 - 9y^2 - 32x - 18y + 19 = 0 \Rightarrow 4(x^2 - 8x + 16) - 9(y^2 + 2y + 1) = -19 + 64 - 9$
$\Rightarrow \frac{(x-4)^2}{9} - \frac{(y+1)^2}{4} = 1$. So, if c is the distance between the center and a focus, we get that $c^2 = 9 + 4 \Rightarrow c = \sqrt{13}$. So the distance between the foci is $2\sqrt{13}$
D. The point of intersection of the asymptotes of any hyperbola is the center of the hyperbola, so
$36x^2 - y^2 + 360x + 6y - 855 = 0 \Rightarrow 36(x^2 + 10x + 25) - (y^2 - 6y + 9) = 855 + 900 - 9$
$\Rightarrow \frac{(x+5)^2}{1746/36} - \frac{(y-3)^2}{1746} = 1$. So the center is at (-5,3).

10. **A.** $2008 = 2^3 \cdot 251$
B. The prime factorization of $6600 = 2^3 \cdot 3^1 \cdot 5^2 \cdot 11^1$. So the number of factors is determined by (3+1)(1+1)(2+1)(1+1)=48.
C. $\left\lfloor \frac{173}{5} \right\rfloor + \left\lfloor \frac{173}{25} \right\rfloor + \left\lfloor \frac{173}{125} \right\rfloor + \left\lfloor \frac{173}{625} \right\rfloor + \left\lfloor \frac{173}{3125} \right\rfloor = 34 + 6 + 1 + 0 + 0 + ... = 41$
D. Notice that 27mod12=3 (since when 27 is divided by 12 there is a remainder of 3), 13mod5=3 and 87mod9=6. So 3+3-6=0. So the smallest natural number, when divided by 6 that leaves a remainder of 0 is A=6.

11. **A.** By multiplying the second equation by 2 we get -14x+6y=-78. Subtracting this equation from the first equation we get 18x=90, which reduces to x = 5. Back substituting we get that $y = \frac{-4}{3}$
B. Use the substitution $u = \frac{1}{x}, v = \frac{1}{y}$ to transform the system to $u - v = \frac{1}{6}$ and $3u + 2v = \frac{13}{6}$. Multiplying the first equation by 2 yields $2u - 2v = \frac{2}{6}$, adding this to the second equation gives $5u = \frac{15}{6} \Rightarrow u = \frac{1}{2}$. Back substituting we see that $v = \frac{1}{3}$. Substituting these values back in to get x and y, we get x=2 and y=3.

C. Notice that the second equation can be rewritten as $(x+y)^2 = 625 \Rightarrow x+y=25$. Also, notice that the first equation can be rewritten in the following way: $(x-y)(x+y)=25 \Rightarrow (x-y)(25)=25 \Rightarrow x-y=1 \Rightarrow x=y+1$. Substituting into the second equation we get $(x+y)^2 = 625 \Rightarrow (2y+1)^2 = 625 \Rightarrow 2y+1=25 \Rightarrow y=12$. Finally with one final substitution we get that x=13.

D. If this is graphed the region turns out to be a triangle with height at the point of intersection of the two lines and base length that is the distance between the x-intercepts. The lines intersect at (-1,4) and the two x-intercepts are (1,0) and $(\frac{-7}{3},0)$. So the area is $\frac{1}{2}(4)(1-\frac{-7}{3})=\frac{20}{3}$.

12. **A.** $\begin{vmatrix} 1 & 4 & 0 \\ -3 & 7 & 5 \\ 2 & -4 & -3 \end{vmatrix} = 1\begin{vmatrix} 7 & 5 \\ -4 & -3 \end{vmatrix} - 4\begin{vmatrix} -3 & 5 \\ 2 & -3 \end{vmatrix} = 1[7(-3)-5(-4)]-4[(-3)(-3)-5(2)]=3$

B. A matrix is invertible if the determinant is equal to 0. So the determinant is 0 when $-6-12m=0 \Rightarrow m=\frac{-1}{2}$

C. Det(2D-4F)=2Det(D)-4Det(F) = 2(4) – 4(-3) = 20

D. We can determine that the original system was $12x-3y=2$ and $11x+y=-3$. Solving this system gives the solution of $(\frac{-7}{45},\frac{-58}{45})$, so the sum of the solutions is $\frac{-13}{9}$.

13. **A.** True, since every function of that form is odd and all will pass the vertical line test.

B. True

C. True, Using order of operations

D. True, for example $f(x)=x^2+1$

14. **A.** $1600_7 = 1(7^3)+6(7^2)+0(7)+0(1)=637_{10}$

B. $1600_{12} = 1(12^3)+6(12^2)+0(12)+0(1)=2592_{10}$

C. $1600_{10} = 4(7^3)+4(7^2)+4(7)+4(1)=4444_7$

D. $1600_{10} = 11(12^2)+1(12)+4(1)=B14_{12}$

15. **A.** $(2x+1)(x-\frac{1}{2})^2 = (2x+1)(x^2-x+\frac{1}{4}) = 2x^3-x^2-\frac{1}{2}x+\frac{1}{4}$

B. Using long division we find the quotient to be $6x^2-2x-18+\frac{9x+53}{x^2+3}$

C. This graph has a double root at x=2 and two imaginary roots. Thus, it touches and/or crosses the x-axis only one time
D. The domain restriction comes from the denominator. Setting the denominator equal to 0 and solving we see that x=0 and x = -4 cannot be in the domain. So the domain is $(-\infty,-4)\cup(-4,0)\cup(0,\infty)$.