Algebra 1 Solutions – January 2008 Invitational

1. Using the Order of Operations:
   $18x-63+12x-4+2x-3=8+3x-28+12+3x \Rightarrow 32x-70=6x-8$
   $\Rightarrow 26x=62 \Rightarrow x=\frac{31}{13}$
2. If the longest side is x, then $2x+12=48 \Rightarrow x=18$. Then the second longest side is one less, or 17 inches.
3. Multiplying the top equation by 2 gives $6x-22y=14$. Adding the top equation to the bottom gives $-22y-14y=14+4 \Rightarrow -36y=18 \Rightarrow y=\frac{-1}{2}$. Substituting this value of y back into either equation gives x = $\frac{1}{2}$. So the solution is $(\frac{1}{2},\frac{-1}{2})$
4. The slope is $\frac{7-3}{-5-2}=\frac{-4}{7}$. Using the point-slope equation we get that
   $(y-3)=\frac{-4}{7}(x-2) \Rightarrow 7y-21=-4x+8 \Rightarrow 4x+7y=29$.
5. $(\frac{s^5w^3}{f^{-6}l^7})^{-1}(\frac{f^2}{w^4})(\frac{s^2l^{-3}s^{-7}}{l^{-6}})=(\frac{f^{-6}l^7}{s^5w^3})(\frac{f^2}{w^4})(\frac{l^3}{s^5})=s^{-10}w^{-7}f^{-4}l^{10}$
6. $\sqrt{(27-19)^2+(-3-(-5))^2}=\sqrt{64+4}=\sqrt{68}=2\sqrt{17}$
7. X-int: 9x – 4(0) = -18. So (-2,0)
   Y-int: 9(0) – 4y = -18. So (0,9/2)
8. $\frac{2}{7}=\frac{x}{150} \Rightarrow x=\frac{300}{7} \approx 42.86$.
9. $7x-2y=-7 \Rightarrow y=\frac{7}{2}x+\frac{7}{2}$. So the slope of the perpendicular line is $\frac{-1}{7/2}=\frac{-2}{7}$.
10. A: 4(0) – y = 7. So A(0,-7). B: 5x + 0 = 10. So B(2,0). So the midpoint is
    $(\frac{2+0}{2},\frac{0+(-7)}{2})=(1,\frac{-7}{2})$
11. $9x-7=6y \Rightarrow y=\frac{3}{2}x-\frac{7}{6}$. So the slope of the perpendicular line is $\frac{3}{2}$. So the equation of the line is $(y-(-3))=\frac{3}{2}(x-2) \Rightarrow y+3=\frac{3}{2}x-3 \Rightarrow y=\frac{3}{2}x-6$
12. The solution to the first inequality is
    $2x-14-5x-20 \ge 3x+3-1 \Rightarrow -3x-34 \ge 3x+2 \Rightarrow -36 \ge 6x \Rightarrow x \le -6$.
    The solution to the second inequality is $\frac{1}{4}x \ge -3 \Rightarrow x \ge -12$.
    So the overall solution is $-12 \le x \le -6$
13. $\frac{3}{6+5+3+1}=\frac{3}{15}=\frac{1}{5}=20\%$

14. 4x7 = (4+1)(7-2)=25. 6x(-1) = (6+1)(-1-2) = -21.
$25\text{star}(-21) = 25^2 + (-21)^2 = 1066 = 1+0+6+6 = 13$. 18star13 = 13-18+1= -4
$\frac{NM}{A} + N - GM - GA + 2 = GN - NI + 3I \Rightarrow \frac{NM}{A} + N - GN + NI = GM + GA - 2 + 3I$

15. $\Rightarrow N(\frac{M}{A} + 1 - G + I) = GM + GA + 3I - 2 \Rightarrow N = \frac{GM + GA + 3I - 2}{\frac{M}{A} + 1 - G + I} = \frac{GMA + GA^2 + 3AI - 2A}{M + A - GA + IA}$

$= \frac{-GMA - GA^2 - 3AI + 2A}{-IA + GA - M - A}$

16. $s = \frac{6}{7}w = \frac{6}{7}(\frac{4}{3}f) = \frac{6}{7}(\frac{4}{3}(\frac{2}{5}l)) = \frac{16}{35}l \Rightarrow \frac{l}{s} = \frac{35}{16}$
17. $y \le 3x - 6$ AND $y \ge x + 1$. When graphed the solution is in quadrant I only.
18. I and IV only since in each of those there is exactly one input for every output.
19. $2^5 \cdot 3^3 \cdot 5^2 \cdot 11$
20. 2007 is divisible by 3, by definition 1 is not prime, 2008 is divisible by 2. Thus only II and V.
21. $(x+8)(x-3)(2-x)(11-9) = (x^2 + 5x - 24)(-2x + 4) = -2x^3 - 6x^2 + 68x - 96$. So $\frac{|-6-(-2)|}{-96-68} = \frac{4}{-164} = \frac{-1}{41}$
22. The solutions are x = 3, -3. So p = 3, q = -3. Then
$(-3-1)(\frac{3^2}{(-3)^2}(3-1)) - 2(-3) + 3 - 1 = (-4)(2) + 6 + 2 = 0$
23. There are 7 regions. Let z be the number of people who ate all 3 desserts. Then we have that
$3 + 6 + 6 + x + x + y + y + z = 30 \Rightarrow 15 + 2(x+y) + z = 30 \Rightarrow 2(6) = 15 \Rightarrow z = 3$
24. The equation of the line is $(y-5) = \frac{-3-5}{7-(-6)}(x+6) \Rightarrow y - 5 = \frac{-8}{13}(x+6)$. If we check all of the statements, they are all true, thus E.
25. $-9x + 21 = 3x - 3 \Rightarrow 12x = 24 \Rightarrow x = 2$. So 6(2+1) – 5 = 13
26. Reducing x^4yz^5. So using the values we get $(2)^4(3)(-1)^5 = -48$
27. They intersect when $x + 1 = 2x + 6 \Rightarrow x = -5$. Substituting we get y = -5+1 = -4. The point (-5,-4) is in quadrant 3. So 5-4+3 = 4
28. The expression becomes
$3 + 3c - c - c^2 = -12 \Rightarrow c^2 - 2c - 15 = 0 \Rightarrow (c-5)(c+3) = 0 \Rightarrow c = 5, -3$. So the sum is 2.
29. $(-4)f(g(3)) = (-4)(f(4)) = (-4)(-1) = 4$
30. The sequence is +0,+1,+2,… So the next 3 numbers are 16,22,29. The sum is 67.