

Pre-Calculus Team Solutions Florida Invitational MIDDLETON TIGERS February 24, 2007
no calculator allowed

$$1. A. \underline{3} \begin{array}{r} 1 \ -1 \ -13 \ 1 \ 12 \\ \underline{ \ 3 \ 6 \ -21 \ -60} \\ 1 \ 2 \ -7 \ -20 \ -48 \end{array}$$

B. $\{1,2,3,4,6,12\} = 28$

C. $\frac{12}{1} = 12$

$$D. \underline{2} \begin{array}{r} 1 \ -1 \ -13 \ 1 \ 12 \\ \underline{ \ 2 \ 2 \ -22 \ -42} \\ 1 \ 1 \ -11 \ -21 \ -30 \end{array}$$

+ - - + + 2 changes, $\frac{-30}{2} = -15$

2. $5|A+7|=30, |A+7|=6, A=-1$ or $A=-13, -14$

$2^{2B+1} = 2^5 = 2B+1=5, B=2$

$\log_2(c^2 - 9) = \log_2 16, c^2 - 9 = 16, c^2 = 25, c = 5$

$(4D)^{\frac{3}{4}} = 27, 4D = 9, D = 2\frac{1}{4}, 2.25$

3. $x^2 - 26x + 169 + y^2 = 169, (x-13)^2 + y^2 = 13^2, A = 26$

$y^2 + 2y + 1 = -4x + 1 - 5, (y+1)^2 = 4(x-1), B = 4$

$9x^2 - 18x + 4y^2 + 16y = 11$

$9(x^2 - 2x + 1) + 4(y^2 + 4y + 4) = 11 + 9 + 16$

$9(x-1)^2 + 4(y+2)^2 = 36$

$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} = 1, c^2 = 5, c = \sqrt{5}$

$25(x^2 - 4x + 4) - 9(y^2 + 8y + 16) = 269 + 100 - 144$

$25(x-2)^2 - 9(y+4)^2 = 225$

$\frac{(x-2)^2}{9} - \frac{(y+4)^2}{25} = 1, a^2 = 9, a = 3, D = 6$

4. Doubles are eliminated from sample space.

A. $(1,4)(4,1)(2,3)(3,2) \frac{4}{30} = \frac{2}{15}$

B. $(1,2)(2,1)(1,3)(3,1)(4,1)(1,4)(1,5)(5,1)$

$(1,6)(6,1) \frac{10}{30} = \frac{1}{3}$

C. $(4,6)(6,4)(5,6)(6,5) \frac{4}{30} = \frac{2}{15}$

D. $(1,2)(2,1)(1,3)(3,1) \frac{4}{30} = \frac{2}{15}$

5. A. $[(2 + 2i)^2]^2 = (4 + 8i + 4i^2)^2 = (8i)^2 = 64$

B. $\frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4}(\frac{5}{12})} = \frac{36 + 20}{48 - 15} = \frac{56}{33}, B = 89$

C. $x^2 + 6x + 9 + 4(y^2 - 2y + 1) = -9 + 9 + 4$
 $(x+3)^2 + 4(y-1)^2 = 4, -3+1 = -2, C = -2$

D. $\frac{2\pi}{\frac{\pi}{4}} = 8$

6. $-\left[\cos\frac{\pi}{2}\cos x - \sin\frac{\pi}{2}\sin x\right]$
 $= -[0 - \cos x - 1 \cdot \sin x] = \sin x$

7. $6 + 3\sqrt{3} - 2\sqrt{6 + 3\sqrt{3}}\sqrt{6 - 3\sqrt{3}} + 6 - 3\sqrt{3}$
 $= 12 - 2\sqrt{36 - 27} = 12 - 2\sqrt{9} = 12 - 6 = 6$

$B^{\frac{1}{6}} = \sqrt{3}$

$B = (\sqrt{3})^6 = 27$

$(\log_2(\log_c 256)) = 3$

$\log_c 256 = 8$

$c^8 = 256, c = 2$

8. A) $2\sin\left(-\frac{\pi}{2}\right) = -2, A = -2$

B) $2\sin\left(3\pi - \frac{\pi}{2}\right) = 2\sin\left(\frac{5\pi}{2}\right) = 2, B = 2$

C) $2\sin\left(\pi t - \frac{\pi}{2}\right) = 0, \pi t - \frac{\pi}{2} = 0, \pi\left(t - \frac{1}{2}\right) = 0,$

$t = \frac{1}{2}, C = .5$

D) period $\frac{2\pi}{\pi} = 2, t = 1, D = 1$

9. A) $M^{-1} = \frac{1}{10}(-4) = \frac{-2}{5}, A = \frac{2}{5}, A = .4$

B) $f(3) = 28, f^{-1}(28) = 3, B = 3$

C) $6(3a)^2(-2b)^2 = 6(9a^2)(4b^2) = 54 \times 4 = 216, C = 216$

D) Algebra books can be arranged in 5! ways. Geometry books in 4! different ways. They can be placed together in 5 different ways.

$5 \times 120 \times 24 = 14400, D = 14400$

10. $r + r + \frac{\theta}{360^\circ} \times 2\pi r = 12,$

$\frac{\theta}{360^\circ} \times \pi r^2 = 8, \frac{\theta}{360^\circ} = \frac{8}{\pi r^2}$

$r + r + \frac{8}{\pi r^2} \times 2\pi r = 12, 2r + \frac{16}{r} = 12$

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$$2r^2 + 16 = 12r, r^2 - 6r + 8 = 0, r = 2, 4,$$

$$\mathbf{A=2, B=4}$$

$$\text{C) } 90 = \frac{1}{2}(18)(15)\sin C, 90 = 135\sin C,$$

$$\frac{90}{135} = \sin C, \sin C = \frac{2}{3}$$

$$\text{D) } 6 + 6 + \frac{1}{12}(12\pi) = 12 + \pi$$

$$11. \text{ A) } 1 = r(\cos \Theta \cos 30^\circ - \sin \Theta \sin 30^\circ)$$

$$1 = r \frac{\sqrt{3}}{2} \cos \Theta - r \frac{1}{2} \sin \Theta$$

$$1 = \frac{\sqrt{3}}{2}x - \frac{1}{2}y, \mathbf{A} = \sqrt{3}$$

$$\text{B) } y = 0, 0 = -1 + 5t, t = \frac{1}{5}$$

$$x = 3 + 2\left(\frac{1}{5}\right) = 3\frac{2}{5}, \mathbf{B=3.4}$$

$$\text{C) } 25(y^2 - 4y + 4) - 9(x^2 + 8x + 16) = 269 + 100 - 144$$

$$25(y-2)^2 - 9(x+4)^2 = 225, \frac{(y-2)^2}{9} - \frac{(x+4)^2}{25} = 1$$

$$m = \frac{-3}{5}, 3x + 5y = 2, c = \frac{-2}{5}, \mathbf{C=.4}$$

$$\text{D) } \frac{\frac{3}{4} - m}{1 + \frac{3}{4}m} = 1, \frac{3 - 4m}{4 + 3m} = 1, 3 - 4m = 4 + 3m, -1 = 7m$$

$$m = \frac{-1}{7}, x + 7y = 37, D = \frac{37}{7}$$

$$12. \text{ A) } 24 = \frac{1}{2}(6)(10)\sin C, 24 = 30\sin C, \sin C = \frac{4}{5}$$

$$\text{B) } c^2 = 36 + 100 - 2(6)(10)\left(\frac{3}{5}\right),$$

$$c^2 = 136 - 72 = 64, \mathbf{c=8}$$

$$\text{C) } \frac{\frac{16\sqrt{3}}{4}}{24} = \frac{\sqrt{3}}{6}$$

$$\text{D) } \frac{1}{2}(4)(4)\left(\frac{4}{5}\right)\left(\frac{1}{24}\right) = \frac{4}{15}$$

$$13. \text{ A) } -2 = 7 + 8d, -9 = 8d, d = \frac{-9}{8}$$

$$a_3 = 7 + 2\left(\frac{-9}{8}\right) = 7 - \frac{9}{4} = 4\frac{3}{4}$$

$$\text{B) } \frac{6}{1 - \frac{2}{3}} = \frac{6}{\frac{1}{3}} = 18$$

$$\text{C) } \frac{\sqrt{2}(\sqrt{2})^{11}}{\sqrt{2}(\sqrt{2})^3} = (\sqrt{2})^8 = 16, \mathbf{16:1}$$

$$\text{D) } 38 = a_1 + (14)(-3), a_1 = 80,$$

$$a_3 = 80 + 2(-3) = 74$$

$$14. \text{ A) } \frac{1}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot 4$$

$$\text{B) } \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{5^3}{6} \cdot 4 + \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{21}{1296} = \frac{7}{432}$$

$$\text{C) } \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{16} = \frac{625}{1296} \quad \text{D) } \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{1296}$$

$$15. \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 13 \\ 5 & 5 \end{bmatrix},$$

$$\text{Inverse: } \frac{1}{7} \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{2}{7} & \frac{1}{7} \\ -\frac{3}{7} & \frac{2}{7} \end{bmatrix} \begin{bmatrix} 1 & 13 \\ 5 & 5 \end{bmatrix} = \begin{bmatrix} 1 & \frac{31}{7} \\ 1 & \frac{-29}{7} \end{bmatrix}$$

$$\mathbf{A=1, B=\frac{31}{7}, C=1, D=\frac{-29}{7}}$$