Calculus Team Questions Sponsor Copy

SOLUTIONS

1. A: $\lim_{x\to -3}(x^2-x-2)=10$

B : $\lim_{x\to 7}\frac{2x-14}{\sqrt{x+9}-\sqrt{2x+2}}\cdot\frac{\sqrt{x+9}+\sqrt{2x+2}}{\sqrt{x+9}+\sqrt{2x+2}}=\lim_{x\to 7}\frac{-2\,\cancel{(x-7)}\cdot\left(\sqrt{x+9}+\sqrt{2x+2}\right)}{\cancel{-x+7}}=-16$

C : $\ln y=\lim_{x\to\infty}x\cdot\ln\left(1+\frac{2}{x}\right)=\lim_{x\to\infty}\frac{\ln\left(1+\frac{2}{x}\right)}{\frac{1}{x}}=2\to y=e^2$

D: $\lim_{x\to 0}\frac{\sin(2x)-2x^2-2x}{x^2}=\lim_{x\to 0}\frac{-2\cos(2x)-4x-2}{2x}=\lim_{x\to 0}\frac{-4\sin(2x)-4}{2}=-2$

2. A: $f'(x)=2\sin(2x)\cos(2x)\cdot 2\to f'\left(\frac{\pi}{6}\right)=\sqrt{3}$ B: $g'(x)=\frac{1}{\sqrt{x^2-1}}+Arc\cos\frac{1}{x}\to g'(\sqrt{2})=1+\frac{\pi}{4}$

C: $h(x)=\sqrt{4-x^2}\to h'(x)=\frac{-x}{\sqrt{4-x^2}}\to h'(1)=-\frac{\sqrt{3}}{3}$ D: $h'(x)=x^{\ln x}\cdot\frac{2\ln x}{x}\to h'(e)=2$

3. A: $-\ln(1-x)\Big]_0^{\frac{1}{2}}=-\ln\frac{1}{2}-\ln 1=-\ln\frac{1}{2}$ or $\ln 2$ B. $\lim_{b\to\infty}\left(\frac{1}{2}\arctan\frac{x}{2}\Big]_0^b\right)=\frac{\pi}{4}$

C: $\frac{1}{2}\ln(1+x^2)\Big]_0^1=\frac{1}{2}\ln 2$ or $\ln\sqrt{2}$ D: $x-\arctan x\Big]_0^1=1-\frac{\pi}{4}$

4. A: Check -4 and local maxima which occur at -3 and 2. $g(-4)=1; g(-3)=3/2; g(2)=\frac{\pi}{2}$ Max is $\frac{\pi}{2}$.

B: Check 4 and local minimum which occurs at 0. $g(4)=\frac{\pi}{2}-\frac{5}{2}$ and $g(0)=0$ Min is $\frac{\pi}{2}-\frac{5}{2}$.

C: $g(-1)=\int_0^{-1}f(t)dt=-\int_{-1}^{0}f(t)dt=\frac{1}{4}$ D: Inflection points occur at -2, 1, and 3. The sum is 2.

5. A: $x\cdot 2y\frac{dy}{dx}+y^2-x^3\frac{dy}{dx}-y\cdot 3x^2=0\to\frac{dy}{dx}=\frac{3x^2y-y^2}{-x^3+2xy}\to\left.\frac{dy}{dx}\right|_{(1,4)}=-\frac{4}{7}$

B: $-x^3+2xy=0\to -x\left(x^2-2y\right)=0\to y=\frac{x^2}{2}$ Substitute into the original to obtain $x=\sqrt[5]{-48}$.

C: $3x^2y-y^2=0\to y(3x^2-y)=0\to y=3x^2\to x=\sqrt[5]{2}$ D: $\frac{dy}{dx}=\frac{3x^2y-y^2}{-x^3+2xy}\to\left.\frac{dy}{dx}\right|_{(1,-3)}=\frac{18}{7}$

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6. A: $\left.\frac{d^2y}{dx^2}\right|_{(1,1/2)} = \frac{2\cdot 2y\frac{dy}{dx}-y^2}{x^2} = \frac{1\cdot\frac{1}{4}-\frac{1}{4}}{1} = 0$:

B: $\int\frac{dy}{y^2}=\int\frac{1}{x}dx \to -\frac{1}{y}=\ln(x)+C.\ \ -2=C \to y=\frac{-1}{\ln(x)-2}$ and $x=e^2$ is the vertical asymptote. Note: $\lim_{x\to 0^+}\frac{-1}{\ln(x)-2}=0.$ C: $\lim_{x\to\infty}\frac{-1}{\ln(x)-2}=0$ so $y=0$ is a horizontal asymptote. D: $f(e)=\frac{-1}{\ln e-2}=1$

7. A: $\left.\frac{4x}{\sqrt{1-x^2}}+4Arc\sin x\right|_{x=1/2}=\frac{4\sqrt{3}+2\pi}{3}$ C: $\lim_{x\to 0}\frac{\frac{1}{x^2+1}}{\frac{1}{\sqrt{1-x^2}}}=1$ D: $\left.\frac{1\cdot 2x}{x^2\sqrt{x^4-1}}\right|_{x=\sqrt{2}}=\frac{\sqrt{6}}{3}$

B: $u=\arctan x; dv=dx \to du=\frac{1}{x^2+1}dx$ and $v=x.$ $x\arctan x\Big]_0^1-\int_0^1\frac{x}{x^2+1}dx=\frac{\pi}{4}-\frac{1}{2}\ln(x^2+1)\Big]_0^1=\frac{\pi}{4}-\frac{1}{2}\ln 2$

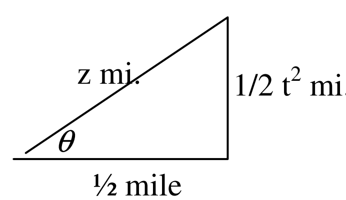
8. A: $A=\frac{1}{2}bh=\frac{1}{2}\cdot 2\cdot 1=1$ B: $A=\int_0^1\frac{x^2}{2}dx+\frac{1}{2}\cdot b\cdot h=\frac{1}{6}+\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{2}=\frac{1}{4}$

C: $\int_0^1\left(\frac{x}{2}-\frac{x^2}{2}\right)dx=\frac{x^2}{4}-\frac{x^3}{6}\Big]_0^1=\frac{1}{12}$ D: $\int_{-4}^0\left(\left(-\frac{3}{2}x+2\right)-\frac{x^2}{2}\right)dx=-\frac{3}{4}x^2+2x-\frac{x^3}{6}\Big]_{-4}^0=\frac{28}{3}$

9. A: $f(0)+f(1)+f(2)+f(3)=0+\frac{1}{2}+2+\frac{9}{2}=7$ B: $2(f(2)+f(4))=2(2+8)=20$

C: $f(1/2)+f(3/2)+f(5/2)+f(7/2)=1/8+9/8+25/8+49/8=84/8=21/2$

D: $1/2\cdot 1\big(f(0)+2f(1)+2f(2)+2f(3)+f(4)\big)=1/2\cdot 22=11$



10. A: $\tan\theta=t^2\to\sec^2\theta\frac{d\theta}{dt}=2t; 2\cdot\frac{d\theta}{dt}=2\cdot 1\to\frac{d\theta}{dt}=1$

B. $\frac{ds}{dt}=t;\left.\frac{ds}{dt}\right|_{t=5}=5\frac{\text{mi}}{\text{min}}=5\cdot 60\text{mph}=300$ C: $\frac{1}{2}^2+(\frac{1}{2}t^2)^2=z^2\to t^3=2z\frac{dz}{dt};1=2\cdot\frac{\sqrt{2}}{2}\frac{dz}{dt}\to\frac{dz}{dt}=\frac{\sqrt{2}}{2}$

D: $v(t)=t\to t=2$. $\tan\theta=t^2\to\sec^2\theta\frac{d\theta}{dt}=2t;17\cdot\frac{d\theta}{dt}=2\cdot 2\to\frac{d\theta}{dt}=\frac{4}{17}$

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11. A: $\pi\int_0^{\ln 2} e^{2x}dx = \pi\cdot\frac{e^{2x}}{2}\Big]_0^{\ln 2} = 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$ B: $\int_0^{\ln 2} e^{2x}dx = \frac{e^{2x}}{2}\Big]_0^{\ln 2} = 2 - \frac{1}{2} = \frac{3}{2}$

C: $\pi\int_0^{\ln 2}\left(4-e^{2x}\right)dx = \pi\cdot\left(4x-\frac{e^{2x}}{2}\right)\Big]_0^{\ln 2} = \pi\cdot\left(4\ln 2 - 2 + \frac{1}{2}\right) = \pi\cdot\left(\ln 16 - \frac{3}{2}\right); 16\cdot\frac{3}{2} = 24$

D: $\int_0^{\ln 2}\left(2-e^{x}\right)^2 dx = \int_0^{\ln 2}\left(4-4e^{x}+e^{2x}\right)dx = 4x - 4e^{x} + \frac{e^{2x}}{2}\Big]_0^{\ln 2} = \ln 16 - 8 + 2 - (0 - 4 + \frac{1}{2}) = \ln 16 - \frac{5}{2}; 16\cdot\frac{5}{2} = 40$

12. A: $v(t) = \frac{3}{2}t^2 - 12t + 18 \to v(1) = \frac{15}{2}$

B: $v(t) = 0 \to t = 2; s(0) = -1; s(2) = 15; s(3) = \frac{25}{2} \to 15$ is the max

C: See part B. $16 + \frac{5}{2} = \frac{37}{2}$ D: $a(t) = 3t - 12 \to a(2) = -6$

13. A: $\lim_{x\to\frac{\pi}{2}^-} = \frac{2\sec^2 x}{\sec x\cdot\tan x} = \lim_{x\to\frac{\pi}{2}^-}\frac{2}{\sin x} = 2$ B: $\ln y = \lim_{x\to 0}\frac{\ln(1+3x)}{2x} = \frac{3}{2} \to y = e^{\frac{3}{2}}$

C: $\lim_{x\to 0}\left(\frac{x-e^{x}+1}{xe^{x}-x}\right) = \lim_{x\to 0}\left(\frac{1-e^{x}}{xe^{x}+e^{x}-1}\right) = \lim_{x\to 0}\left(\frac{-e^{x}}{xe^{x}+e^{x}+e^{x}}\right) = -\frac{1}{2}$

D: $\ln y = \lim_{x\to 0^+}\frac{\ln(e^{x}-1)}{\frac{1}{x}} = \lim_{x\to 0^+}\frac{e^{x}\cdot -x^2}{e^{x}-1} = \lim_{x\to 0^+}\frac{e^{x}\cdot -2x + e^{x}\cdot -x^2}{e^{x}} = 0 \to y = 1$

14. A: $\left.\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right|_{t=1} = \left.\frac{1}{2t}\right|_{t=1} = \frac{1}{2}$ B: $\frac{\frac{d}{dt}\left(\frac{1}{2t}\right)}{4t} = \frac{\frac{-1}{2t^2}}{4t} = \frac{-1}{8t^3} = -\frac{1}{8}$ at $t = 1$.

C: $\left.\sqrt{(4t)^2+4}\right|_{t=1} = \sqrt{20} = 2\sqrt{5}$ D: $\frac{dy}{dx} = \frac{1}{2} = \tan\theta \to \sin\theta = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

15. A: $\frac{dP}{dt} = 0.6y\left(1-\frac{y}{500}\right) \to P(t) = \frac{500}{1+49e^{-0.6t}} \to \lim_{t\to\infty}P(t) = 500$

B: The population grows the fastest at 1/2 the carrying capacity or 250. C: In logistics growth, the growth rate approaches 0 as time increases. 0

D: $250 = \frac{500}{1+49e^{-6t}} \to 1 = \frac{2}{1+49e^{-0.6t}} \to 1 + 49e^{-0.6t} = 2 \to e^{-0.6t} = \frac{1}{49} \to t = \frac{\ln 49}{\frac{3}{5}} = \frac{10\ln 7}{3}; a\cdot b = 30$