Solutions

1. A

$\overset{\frown}{CE}$ = 130 degrees by definition of central angle, since $\triangle AGB$ is an isosceles triangle.
$\angle CDE = 65$ by definition of inscribed angle. So the supplement is 180-65=115.

2. D

$M = 2D \quad M - 10 = 3(D - 10) - 1$
$2D - 10 = 3D - 31$
$D = 21, M = 42. \quad D + M = 21 + 42 = 63$

3. D

$$\frac{1+\frac{1}{1-\frac{x}{y}}}{1-\frac{3}{1-\frac{x}{y}}} \rightarrow \frac{\frac{y-x}{y-x}+\frac{y}{y-x}}{\frac{y-x}{y-x}-\frac{3y}{y-x}} \rightarrow \frac{\frac{2y-x}{y-x}}{\frac{-2y-x}{y-x}} \frac{2y-x}{-2y-x}$$

Which is equivalent to $\frac{x-2y}{x+2y}$.

4. C

$(i+1)^{10} \rightarrow \left((i+1)^2\right)^5 \rightarrow (2i)^5 = 32i$

5. D

x is the largest side of the 5-70-105; however, the smallest side of the 5-75-100. B is the largest side of the 5-75-100; however, the smallest one of the 35-50-95. So the largest side is F, since it is across from the largest angle (95).

6. C

$\left(\frac{K}{4}\right)^2 = 2 \rightarrow K^2 = 16 \bullet 2 \rightarrow K = \pm 4\sqrt{2}$

7. A

The ratio in a geometric progression is the quotient of a_{n+1} and a_n.

So $r \bullet B \rightarrow \frac{B}{A} \bullet B = \frac{B^2}{A}$

8. C

$\left(x^2 - 2y\right)^7 \rightarrow (1-2)^7 \rightarrow (-1)^7 \rightarrow -1$

9. C

$9^2 = 3(3+x) \rightarrow 81 = 9 + 3x \rightarrow 72 = 3x \rightarrow x = 24$

10. B

The sum of every three-pair angle and the unmarked angle is equal to 360.
So the sum of all the letters is equal to $360(6) - 180(6-2) = 1440$

11. D

By the definition of a median in a right-triangle, its measure is half of the length of the hypotenuse. The 9-12-15 triangle has a median of $\frac{15}{2}$. Twice $\frac{15}{2}$ is 15.

12. D

$$\sqrt{6-\sqrt{6-\sqrt{6-\sqrt{6-\sqrt{...}}}}} = x \rightarrow 6-x = x^2$$

$x^2 + x - 6 = 0 \rightarrow (x+3)(x-2) = 0.\ x = 2,$ becasue a $\sqrt{...}$ cannot be negative.

13. A

$\sum_{k=1}^{11} i^k$ every 8 terms the sum goes to zero. The pattern is $i, -1, -i, 1$.

So the sum of the first eleven terms would be -1.

14. B

(This is a MEAN one!!) R = inscribed, r = Circumscribed.

$R = \frac{2A}{a+b+c}$ (where a, b and c are the sides and A is the area of the triangle)

$r = \frac{abc}{4A}$ $\qquad R \div \frac{1}{r} = Rr = \frac{2A}{a+b+c} \cdot \frac{abc}{4A} = \frac{abc}{2(a+b+c)}$

$$Rr = \frac{5 \cdot 7 \cdot 8}{2(5+7+8)} = 7$$

15. C

$(1 + i)\ (3 - 3i)\ (2 - 2i^2)\ (-4 - 4i^2)$

$(-4 - 4i^2) \rightarrow (-4 + 4) \rightarrow 0$. The product is zero.

16. C

$4x^2 + 8y^2 + 24x - 16y - 4 = 0 \rightarrow \frac{(x+3)^2}{12} + \frac{(y-1)^2}{6} = 1$

The area of the ellipse is $ab\pi \rightarrow \sqrt{12} \bullet \sqrt{6} \bullet \pi \rightarrow 6\pi\sqrt{2}$

17. A

$360 - (90 + 108) = 162$

$\frac{180(n-2)}{n} = 162 \rightarrow -360 = -18n \rightarrow n = 20.$ $\quad n - 1 = 19$

18. B

$g(x) = f(h(x)) = 4x^2 - 3 \rightarrow x = 4y^2 - 3 \rightarrow \frac{x+3}{4} = y^2$

$g^{-1}(x) = \pm\frac{\sqrt{x+3}}{2} \quad \rightarrow \quad \left|g^{-1}(0)\right| = \frac{\sqrt{3}}{2}$

19. B

$p = \frac{r}{2\pi r} \rightarrow \frac{1}{2\pi} \qquad g = 2\pi r$

$pg = \frac{2\pi r}{2\pi} \rightarrow r = \sqrt{125} = 5\sqrt{5}$

20. A

$y = 3x - 2,$ when x=1, y=1.

$y - y_1 = \frac{-1}{3}(x - x_1) \rightarrow y = \frac{-1}{3}x + \frac{4}{3}$. The product of the slope and the y-intercept is $\frac{-1}{3} \bullet \frac{4}{3} = \frac{-4}{9}$

21. C

$(A + B + C + D + E) - (F + G + H + I + J) \rightarrow (180) - (540) = -360$

22. A

$729 @ (4 * -3),\ \left((\pm 2)^2 * (-3)\right) = 2(\pm 2) - (-27) \rightarrow so\ 31\ or\ 23.$

$729 @ 23 = 729^{\frac{1}{6}} = (3^6)^{\frac{1}{6}} = 3 \qquad 729 @ 31 = 729^{\frac{1}{14}} = (3^6)^{\frac{1}{14}} = 3^{\frac{3}{7}} \qquad 3^1 > 3^{\frac{3}{7}}$

$\therefore$ 3 gives a greater absolute value.

23. C

$125 = 5^3 \rightarrow (5)^{3 \bullet \frac{-2}{3}} = 5^{-2} = \frac{1}{25}$

24. C

$\sqrt{\frac{-1(x^2 - 324)}{(x^2 - 324)}} = i$

25. B

$2x^2 + 3x - 1$ has a vertex at $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) \rightarrow \left(\frac{-3}{4}, \frac{-17}{8}\right)$

26. C

$m\overline{OZ} + m\overline{OR} + m\overline{OM} =$ The altitude of the equilateral triangle. so $\sqrt{3}$

27. C

The product of the roots is $\frac{-z}{a}$, where z is the constant and a is the leading coeffiecient.

$2x^3 - 7x^2 - 6x + 15 \rightarrow \frac{-15}{2}$

28. B

$F(x) = \sqrt{x - 13}$ *and* $G(x) = 41x^7 + 13x^6 - 36x^5 + 54x^4 - 78x^3 + x^2 - 11x + 13x^0,\ k = 13$

$G(F(G(F(G(F(13)))))) = G(F(G(F(G(0))))) = G(F(G(F(13)))) = G(F(G(0)))$

$G(F(13)) = G(0) = 13$

29. A

$\sum_{p=1}^{40} p = \frac{n(a_1 + a_n)}{2} \rightarrow \frac{40(41)}{2} = 820$

30. C

$7(30) + 12 + \frac{(60 - 47)}{60}(30) \rightarrow 222 + \frac{13}{2} = \frac{457}{2}$

$\frac{457}{2 \bullet 360} \bullet 12\pi = \frac{457\pi}{60}$