Middleton Invitational	February 2006
Precalculus Team (no calculator)	**Question # 1**

Each of the following statements has a value, indicated by the number in brackets.
Find the sum of the values of the true statements.

[1] Zero is a prime number.

[2] The ellipse with equation $\frac{(x+2)^2}{4}+\frac{(y+3)^2}{5}=1$ has area 20π .

[4] The distance between the three-dimensional points (5, 4, 2) and (3, 6, 1) is 3.

[8] If $f(x)=2\left(3x^{\frac{2}{3}}-5x\right)$, then $f(3)=-15+3\sqrt{3}$.

[16] The vector $<1,1>$ is a unit vector.

Middleton Invitational	February 2006
Precalculus Team (no calculator)	**Question # 2**

A = the greatest solution to the equation $15-6x=x^2+24$

B = the radius of the circle with equation $x^2+62=87-y^2$

C = $\log_4 256$

D = $\sec^2\left(\frac{\pi}{4}\right)$

Find: $\left(\frac{\text{A}-\text{B}}{\text{C}+\text{D}}\right)^{-2}$

Middleton Invitational	February 2006
Precalculus Team (no calculator)	**Question # 3**

Let $f(x) = \frac{x+3}{x-4}$.

A = $f\left(\frac{3}{2}\right)$ **B** = $\lim_{x \to -4} \left(f(x)\right)$ **C** = $f^{-1}(0)$

D = the y-intercept of the graph of $f(x)$

Find: $\text{CD} \div \text{AB}$.

Middleton Invitational	February 2006
Precalculus Team (no calculator)	**Question # 4**

Let $f(x) = g(2x-1)$.

A = the period of f, when $g(x) = \tan x$

The equation $\boldsymbol{x} = \mathbf{B}$ is the vertical asymptote of f, when $g(x) = \frac{1}{3x+4}$

The interval $(\mathbf{C}, \infty)$ is the domain of f, when $g(x) = \log_7 x$

Find: $\frac{120\text{A}}{\pi BC}$.

Middleton Invitational **February 2006**
Precalculus Team (no calculator) **Question # 5**

$f(x) = ax^5 + bx^4 - ax^3 - bx^2 + ax + b$, where a and b are positive integers.

Let **M** be the greatest real root of $f(x)$.

Let **N** be the leading coefficient of $f\left(-\frac{x}{a}\right)$.

Let **P** be the y-intercept of $\frac{f(x)+b}{a}$.

Let **Q** be $f(-1)+f(1)$.

Express the product **MNPQ** in terms of a and b without the use of negative exponents.

Middleton Invitational **February 2006**
Precalculus Team (no calculator) **Question # 6**

Write the domain of each function given below in interval notation.

$f(x) = \sqrt{4-x}$ $\quad g(x) = \frac{4}{x}$ $\quad h(x) = \sqrt[3]{x^3 - 64}$ $\quad j(x) = \frac{1}{x^2 - 16}$

Let **A** be the number of times you wrote the union $(\cup)$ symbol.

Let **B** be the number of functions that include 4 in their domain.

Let **C** be the number of times you wrote $-\infty$.

Let **D** be the number of functions that include 0 in their domain.

Find **A + 10B + 100C + 0.1D** .

Middleton Invitational	**February 2006**
Precalculus Team (no calculator)	**Question # 7**

A = $\cos\theta$**,** if θ is the angle between the vectors <8,15> and $<-8,6>$

B = $\sin\theta$**,** if θ is the smallest angle in a right triangle with legs measuring 20 and 48.

C = $\cot\theta$**,** if $\sec\theta = \csc(2\theta)$ and $0<\theta<\frac{\pi}{2}$.

D = $\tan\theta$**,** if θ is the acute angle between the *x*-axis and the line with equation $x-3y=4$.

Find the product **ABC^{2}D.**

Middleton Invitational	**February 2006**
Precalculus Team (no calculator)	**Question # 8**

Each of the nine letters in the word **MIDDLETON** is written on a ball and placed in a bag.

W = the probability that two balls selected at random, without replacement, both have vowels on them.

X:Y = the odds that two balls selected at random, with replacement, have the same letter.

Z = The number of distinct linear arrangements of all the letters in **MIDDLETON**.

Find $\frac{WZ}{X^2Y}$.

Middleton Invitational **February 2006**

Precalculus Team (no calculator) **Question # 9**

A = $\lim_{x\to 0}\frac{-\cos x+\tan x+1}{x}$

B = $\lim_{x\to 2}\frac{3x^3+x-6x^2-2}{4x^2-13x+10}$

C = $\lim_{x\to 3}(x)$

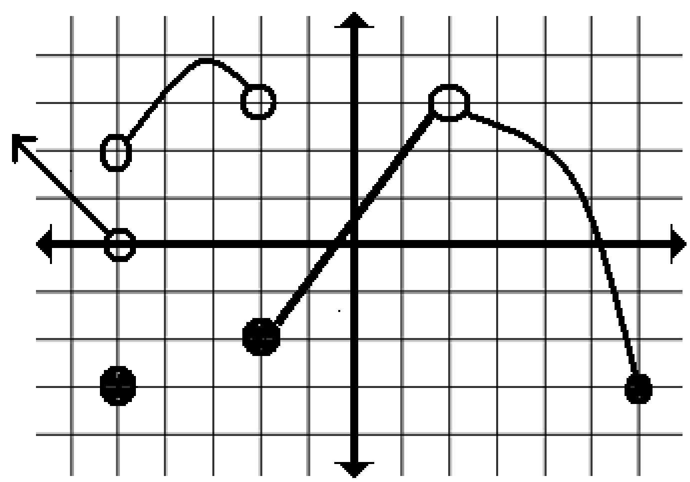
D = $\lim_{x\to\infty}\left(x^{-1}\right)$

Find: **4A + 3B + 2C + D**

Middleton Invitational **February 2006**

Precalculus Team (no calculator) **Question # 10**

The function $f(x)$ is graphed to the right.
Marked points have integral coordinates.
Increments on both axes are 1.



A = The number of values of x at which $f(x)$ is discontinuous on the interval $(-\infty,0)$

B = $\lim_{x\to -5^+} f(x)$

C = $f(-2)$

D = The number of solutions to the equation $f^{-1}(x)=2$

E = The number of solutions to the equation $\frac{f(x)}{2}=1$

Find: **A + 3B + 5C + 7D + 11E**

Middleton Invitational	February 2006
Precalculus Team (no calculator)	Question # 11

Given that 3 is a root of the polynomial $b(x) = 10x^3 - 11x^2 - 72x + 45$:

Let **F** be the least root of *b*.

Let **G** be the greatest root of *b*.

Let **H** = $b(0.2)$.

Find: $H \cdot F^2 + 2G$.

Middleton Invitational	February 2006
Precalculus Team (no calculator)	Question # 12

The polar point $\left(-6, \frac{11\pi}{3}\right)$ has rectangular coordinates **(A, B)**.

The vertex of the parabola with polar equation $r = \frac{1}{3 - 3\cos\theta}$ has rectangular coordinates **(C, D)**.

The ellipse with equation $x^2 + 4y^2 - 4x + 8y = 0$ has eccentricity **E**.

Find: **2A + 4B + 6C + 8D + 10E .**

Middleton Invitational Precalculus Team (no calculator)	February 2006 Question # 13

Find $\sin^2(0.5\theta)\cdot\cos(-2\theta)\cdot\csc^3\theta\cdot\cot(0.5\pi-\theta)$, when $\cos^2\theta = 0.36$ and θ is an angle in standard position with its terminal side in Quadrant I.

Middleton Invitational Precalculus Team (no calculator)	February 2006 Question # 14

Let the vectors $A = <3,-1>$ and $B = <-3,2>$.

$2A-4B = <m,n>$

A is orthogonal to the vector $<5,\ p>$.

B is parallel to the vector $<5,\ t>$.

Find: $m+n-\dfrac{p}{t}$.

Middleton Invitational	**February 2006**
Precalculus Team (no calculator)	**Question # 15**

Find the number of distinct solutions to the equation $(\cos x)(\sin x) = 0$ on the interval $[0,10]$.